In the last class we have seen that the op-amp has in its circuit basically a differential amplifier and we have actually cascaded stages of differential amplifiers in the op-amp and what we get at the output of an op-amp is a very high gain amplified signal but the magnitude of this output signal is basically dependent on the two input signals that are being applied at the two terminals of the op-amp that is the inverting and non-inverting terminal. If the difference between the two signals applied at the two inputs of the op-amp is high then we get a high output voltage but if the difference between the two input signals is less then we get an attenuated output and if the two signals are equal then we are supposed to get a zero output from the op-amp. So the output voltage of an op-amp will be dependent basically upon the difference between the two input signals.

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For example if we have $V_{i1}$ and $V_{i2}$ at the non-inverting and inverting terminals of an op-amp then the output voltage from the op-amp will be highly amplified if this $V_{i1}$ and $V_{i2}$ are opposite and if they are same then it will be only slightly amplified and ideally we are suppose to get zero when they are equal or same voltages at the two terminals. So the overall amplification or overall operation of an op-amp is to reject the common signal and amplify the difference signal. If we have the difference between the two signals higher then we have an amplified output and the common signal which is present at the two input terminals will be reduced to zero. That is why if we consider the noise which is common to both the input signals being applied at the two input terminals of an op-amp, naturally the noise will be attenuated. That is one big advantage which we have in op-amp that the noise part in the two input terminals for the signals being applied will be attenuated or rejected and the difference between the signals at the input terminals is amplified.

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Now let us find out the output voltage when you have two input signals $V_{i1}$ and $V_{i2}$ at the two input terminals of the op-amp. These two signals in general if we consider they may have both in-phase and out-of-phase components. Some portion of the two input signals may be common to both of them and some will be out-of-phase. The resulting output of
the op-amp if we consider as \( V_0 \), which is the output voltage from the op-amp, it is given by \( A_dV_d + A_cV_c \) where the \( A_d \) term is denoting the differential gain of the amplifier and \( A_c \) term is denoting the common mode gain of the amplifier. These two terms are very important when we consider the output voltage or amplified output when we find out, these two play the role for finding out the magnitude. As this \( A_d \) is the differential gain and \( A_c \) is the common mode gain, these two will determine the output component, which is arising because of the difference between the two signals and the common part of the two signals.

The \( V_d \) voltage is denoting the difference voltage between the two signals \( V_{i1} \) and \( V_{i2} \) that we are applying. So \( V_{i1} \) minus \( V_{i2} \) is the difference between the two voltages and a common voltage is denoted by \( V_c \). So \( V_c \) is the average between the two voltages; half of \( V_{i1} \) and \( V_{i2} \). Half of \( V_{i1} \) plus \( V_{i2} \) is the common signal or the common voltage between the two signals. Then we have the output voltage given by this expression of \( V_0 \) equal \( A_dV_d + A_cV_c \).

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![Common Mode Rejection Ratio (CMRR)](image)

The term which is very important in an op-amp is common mode rejection ratio or CMRR. Basically CMRR is the ratio between the magnitude of the two terms that we
have now described, $A_d$ and $A_c$ differential gain and common mode gain of the op-amp. The ratio between the magnitude of the difference gain and the common mode gain of the op-amp is known as CMRR or common mode rejection ratio. In absolute quantity it is this expression, which is the ratio between the magnitudes of these two gains $A_d$ and $A_c$ or it is also taken in logarithmic unit that is log of CMRR is also taken. In that logarithmic unit it will be described as 20 log of magnitude $A_d$ by magnitude $A_c$ to the base 10. In these two ways CMRR is expressed.

Now we look back into that expression of the output voltage again. The output voltage $A_d V_d$ plus $A_c V_c$ we do a little manipulation taking this $A_d V_d$ term common. If we take $A_d V_d$ common, then within bracket we get 1 plus $A_c V_c$ by $A_d V_d$. Then it can be written as $A_d V_d$ into 1 plus $A_c$ by $A_d$ into $V_c$ by $V_d$. The purpose of manipulating in this way is to introduce the term CMRR. That is further made equal to the expression $A_d V_d$ into 1 plus if we look into this term $A_c$ by $A_d$ it is nothing but 1 by CMRR because CMRR is equal to magnitude $A_d$ by magnitude $A_c$. From this we get 1 by CMRR equal to magnitude $A_c$ by magnitude $A_d$. We are replacing this $A_c$ by $A_d$ by 1 by CMRR into $V_c$ by $V_d$. In this expression for the output voltage if we observe closely we find that if CMRR is very high, the value of this common mode rejection ratio for an op-amp is very high, then the output voltage $V_0$ will be almost equal to $A_d V_d$ because this term being high means the overall term 1 by CMRR into $V_c$ by $V_d$ will be very small approaching zero because the denominator is very high. This whole term will approach zero that means we are left with $V_0$ equal to $A_d V_d$.

That is a very significant analysis because it tells that for a high CMRR, op-amp output voltage is basically the difference in gain multiplied by the difference in the input voltages, so the common mode part is almost rejected to zero. That is why we have to have a high CMRR op-amp which will amplify the output signal based on the difference between the two input signals and whatever a common part is almost equal to zero.
For this example you have to determine the output voltage of an op-amp for input voltages $V_{i1}$ is equal to 150 microvolt and $V_{i2}$ is equal to 140 microvolt. In this op-amp, the two voltages which are applied at the non-inverting and inverting terminals are this is $V_{i1}$ and this is $V_{i2}$. They have the magnitudes of 150 microvolt and 140 microvolt. The amplifier has a differential gain $A_d$, 4000 and the value of the CMRR is given as 100. We have to find out the output voltage. As per the descriptions just now we have seen $V_o$, output voltage is given by the expression $A_d$ into $V_d$ within bracket $1 + \frac{1}{CMRR} V_c$ by $V_d$. In this expression, we can plug in the given values for the terms. Let us see the values given; $V_{i1}$ is given, $V_{i2}$ is given and the differential gain is given. Common mode gain is not given but we know that CMRR value is 100. If we know the CMRR value and $A_d$ value that is sufficient because the CMRR is nothing but $A_d$ by $A_c$.

In order to find out the $V_o$, that is the output voltage let us now plug in the values into these expressions. $A_d$ is 4000, we know it. What is $V_d$, the difference voltage we can find out. $V_{i1}$ minus $V_{i2}$ that is the difference voltage; 150-140 and that is 10 microvolt. So 10 microvolt is the difference voltage applied between the two inputs of the op-amp and CMRR is 100, which is given and $V_d$ we have found out.
Vc we can find out because Vc is the common mode part of the two signals and Vc is found out by average between the two signals half of Vi1 plus Vi2. That is equal to half of 150 + 140; both are in microvolt and that gives the value of 145. 145 microvolt is the common mode part of the two signals and the difference component of the two signals we have found out to be 10 microvolt. We can find out now the output voltage V0. According to the expression which we know Ad into Vd into 1 plus 1 by CMRR into Vc by Vd, every component of this expression we know. Now we can plug in, so what is Ad? Ad is equal to 4000; Vd we have found out to be equal to 10 microvolt. The units are written in microvolts. So I need not repeatedly write the units; all are in microvolts. So 1 plus 1 by CMRR is 100. So 100 into Vc is 145, Vd is 10.

Simplifying this expression or finding out the numerical value we now find out what will be this value? 4 into 10 to the power 4 within bracket the terms; denominator is 1,145 by 1000 is 0.145. So 1.145 into 4 into 10 to the power 4 gives the value of 45800 microvolt. To check this value it should come 45,800 microvolt, meaning it is equal to 45.8 millivolt. So 45.8 millivolt output we obtained for this op-amp having the two input signals which are 150 microvolt and 140 microvolt.
Now let us consider an ideal op-amp. You call an op-amp an ideal op-amp because if we consider an op-amp circuit what is inside that circuit is differential amplifier, but they are in stages.

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If we consider the op-amp having input resistance, output resistance and its gain, say $A$ then a simple circuit can be used to explain that op-amp. Here it is a simple circuit having input voltage between these two terminals. This is say 1, 1 dash. These are the input terminals and the input voltage which is applied to the op-amp is a difference input $V_1$ minus $V_2$. $V_1$ and $V_2$ are the two inputs at the two terminals of the op-amp, non-inverting and inverting terminal. Then the difference voltage will be $V_d$, which is equal to $V_1$ minus $V_2$ and the op-amp will have an input resistance and this input resistance is very high and let us denote it by $R_i$. This $R_i$ will represent the input resistance. So the input side is having these two parameters, one is the difference input. This is the input voltage, it has one parameter which is input resistance and what will be the output of the op-amp? Gain multiplied by the difference input into $V_d$.

So $A$ into $V_d$ is the voltage source at the output meaning that we are getting an output voltage $A$ times $V_d$ and this is the voltage source which is denoting that output voltage
which we obtained at the output. But there is output resistance also. Although this value is very small, we cannot ignore and this is $R_0$; that $R_0$ is the output resistance of the op-amp. Finally what we get at the output is $V_0$ which is $AV_d$ minus this drop occurring in the resistance $R_0$. Practically this is the circuit which is used to explain the op-amp. We are basically having a simple circuit representing the op-amp in terms of input resistance, output resistance and the gain of the op-amp. These are the parameters which practically we have in an op-amp and which can be known from the data sheet available for an op-amp also.

This circuit is a practical circuit of the op-amp but we will now consider an ideal op-amp where we will consider that the input resistance $R_i$ is infinite, output resistance $R_0$ is zero and the gain $A$ is also infinite. The ideal op-amp consideration is useful for analyzing the circuits having the op-amp and in our analytical expressions which we will be using or in our analysis of the op-amp circuits it is obtained simply or easily by considering the ideal op-amp analysis. That is why we are imagining an ideal op-amp having the requisites of the parameters with certain specification like as we have now told that $R_i$ should be infinite, $R_0$ should be zero. In this way ideally if these parameters have certain values we consider that op-amp to be an ideal op-amp and in all our analysis from now onwards we will be using ideal op-amp analysis having these properties. So what are the properties of the ideal op-amp or what are the parameter values it should have?
One is that ideal op-amp has the characteristics of first input resistance, $R_i$ should be infinity, output resistance $R_0$ should be zero, voltage gain should be infinite and bandwidth should be infinity. Bandwidth means the characteristics of the op-amp like voltage gain, should not change with very low or very high frequencies; that means throughout the whole frequency range of application the characteristics of the op-amp should not change. For example if voltage gain is $A$ or infinity, it should be same for all the frequency range, whole frequency range. Because we have seen earlier in the case of transistor, voltage gain if we consider for very low frequencies and very high frequencies the gain drops off or reduces; only in the mid band the gain becomes constant. So, that is not a constant gain. It is varying with frequency and that type of analysis or that type of characteristics should not be there for this ideal op-amp; so ideal op-amp will have an infinite bandwidth. That is within that infinite frequency range the characteristics are same. That means we are not having any changed characteristics like changed voltage gain for frequency range; infinite means, practically for whole frequency range we are getting the same characteristics. That is why it is written that bandwidth is infinite.

Perfect balance also is one important property which has to be obeyed by this op-amp. That is $V_0$ is equal to zero when $V_1$ is equal to $V_2$. If we consider the op-amp, one is non-
inverting and one is inverting terminal. Let us have these voltages at the inverting and non-inverting terminals $V_1$ and $V_2$. If the output voltage is zero, $V_0$ is equal to zero, $V_1$ must be equal to $V_2$ and that means that the perfect balance is there for the op-amp. This perfect balance will come only because the differential amplifier which is the basic circuit inside the op-amp has the transistors which are perfectly matched. When the two transistors are perfectly matched or balanced we do not have any unbalanced voltage and that means when these two voltages $V_1$ and $V_2$ are equal then there should not be any output voltage, output voltage should be zero; that is to be maintained. For op-amp that is one criterion.

Another criterion is that characteristics do not drift with temperature. Temperature should not have any effect on the characteristics change. That is whatever characteristics we are getting for the op-amp, may be the voltage gain or other parameters they will not be dependent on temperature. Even if temperature changes we should not have a varying characteristics. All these properties are to be satisfied for the op-amp when we call that op-amp to be ideal op-amp. Actually this ideal op-amp consideration, make our analysis very easier.

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If we consider ideal op-amp $R_i$ is infinity. So, let us consider this op-amp having the non-inverting voltage $V_1$ and inverting terminal has a voltage $V_2$. Difference voltage is then $V_1$ minus $V_2$. That is the voltage between the two terminals. $V_1$ minus $V_2$ is the difference voltage $V_d$ and output voltage is equal to $V_0$. This ideal op-amp has the consideration or the parameter $R_i$ infinite, input resistance is infinity. If there is an input resistance here suppose $R_i$ and that is infinity; so if $R_i$ is infinity there cannot be any current entering into the op-amp. The first consequence of this $R_i$ being infinity is that the current entering into an op-amp is zero. This is an important inference which we will use in all our further analysis. This is the first principle which you will have to use when analyzing an op-amp circuit. That is ideal op-amp consideration has the current entering into the op-amp as zero. As there is no current, there is no current entering into the op-amp.

Another characteristic of an ideal op-amp is that gain is infinity. If input voltage is the difference voltage $V_d$, output voltage is $V_0$, this is the gain of the op-amp which is $A$. As $A$ is infinity for an ideal op-amp and $V_0$ is finite, we are getting a finite voltage at the output. So, what will happen to the $V_d$ value? $V_d$ is equal to $V_0$ by $A$. $V_0$ by $A$ again becomes $V_0$ by infinity which is equal to zero. What it means is that for ideal op-amp, the difference voltage is zero means $V_1$ minus $V_2$ is equal to zero. That means $V_1$ is equal to $V_2$. For the ideal op-amp, the voltage at these two terminals inverting and non-inverting, this is $V_1$, this is $V_2$, these two are equal and these two principles of the op-amp that means one is the current entering into the op-amp is zero and $V_1$ is equal to $V_2$ will be extensively used in all our further analysis. We will be using these two and find out basically what we will be getting at the output voltage.
Remembering these two basic principles of ideal op-amp, let us now consider some practical op-amp circuits. The practical op-amp circuits are those circuits which we very often use and one very simplest application of op-amp is inverting amplifier. What is there in an inverting amplifier? This is an op-amp and we are applying a voltage $V_1$ at the inverting terminal. Inverting terminal is this one which is having a negative symbol. We are having a voltage $V_1$ and we are having resistance $R_1$ here in the input side and $R_f$ is a feedback resistance. It is called feedback because it is from the output to the input. So $R_f$ is called the feedback resistance. This is the circuit of an inverting amplifier using the op-amp. What will be the output voltage $V_0$?

In order to find out the output voltage $V_0$, we will be using the properties of an ideal op-amp. That means we will be using the ideal op-amp considerations. For all these analysis which we are going to do now will be following ideal op-amp properties. The properties are mainly two; one is that there is no current entering the op-amp and $V_1$ equal to $V_2$. $V_1$ equal to $V_2$ means the voltage at the inverting terminal is equal to voltage at this non-inverting terminal. But here we are applying a voltage at $V_1$ through a resistance $R_1$ which is connected to the non-inverting terminal. We will be using the first principle that there is no current entering into the op-amp because of the infinite input resistance. That
means what? The current which is flowing from \( V_1 \), let us name it by \( i_1 \). This current cannot go and enter into op-amp. So, where it will go? It will be following this part through \( R_f \). That means the current coming from \( V_1 \) will be following through the part \( R_f \) and it will not enter into op-amp. If this is so, now we can very easily apply the Kirchoff’s voltage law and find out what is \( V_0 \)? That is what we are going to do.

Using the Kirchoff’s voltage law in this circuit, what is Kirchoff’s voltage law? This \( V_1 \) minus \( i_1 \) into \( R_1 \) minus \( i_1 \) into \( R_f \) minus \( V_0 \) equal to zero. We can write in another way also. What is this loop? If we consider this loop \( V_1 \) minus \( i_1 R_1 \) equal to zero because this point and this point has same voltage and it is grounded. If I consider this loop, we can apply Kirchoff’s voltage law in this loop. Also it is a closed loop.

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It is coming from plus terminal of \( V_1 \) to ground and minus of the \( V_1 \) is also grounded. So this is a closed loop. If we apply this Kirchoff’s voltage law to this loop, it will become \( V_1 \) minus \( i_1 \) into \( R_1 \) and there is no other drop that is equal to zero because this voltage, this voltage is grounded; these are the same voltage. Another loop is starting from this ground, we can go like this in the ground and we can go like this and complete the loop. That means we can start our travel from the ground. Then what will it be? It will be
minus $i_1$ into $R_f$ and then minus $V_1$ equal to zero. So there are two equations which we will be taking help of in order to find out $V_0$. First is $V_1$ minus $i_1 R_1$ zero. That gives us an expression for $i_1$ which is equal to $V_1$ by $R_1$.

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The other equation which we have written from this loop is zero. You can start from ground and write this voltage as zero. This is nothing but the ground voltage is zero minus $i_1 R_f$ minus $V_0$ equal to zero. So what is $V_0$? Minus $i_1$ into $R_f$ and minus $i_1$, we can replace from here. $i_1$ is $V_1$ by $R_1$. So it becomes minus $V_1$ by $R_1$ into $R_f$. So $V_0$ equal to minus $V_1$ by $R_1$ into $R_f$. From that we get the gain $V_0$ by $V_1$. What is that gain? Minus $R_f$ by $R_1$. So this is the gain of an inverting amplifier using op-amp. The inverting op-amp circuit gives the gain minus $R_f$ by $R_1$. That is equal to $V_0$ by $V_1$; that is the inverting op-amp.
If we consider a non-inverting amplifier, as the name non-inverting suggests the voltage will be applied through the non-inverting terminal. This plus terminal is denoting the non-inverting terminal, we are connecting a voltage signal \( V_1 \) and there these two resistances \( R_1 \) and \( R_f \) are connected like this. We are to find out the voltage \( V_0 \). In order to find out the voltage \( V_0 \) again we will be proceeding in the same way and let us denote the current \( i \), by this current which is flowing in the \( R_f \) that is starting from this point. This direction you can choose in any way. Intuitively, I am choosing the direction from right to left because this is the ground. So it will start and end at ground. It will start from here and end at ground but there is no hard and fast rule because we are choosing the direction of current and we can choose it from left to right also; that will also give you ultimately the same result. But I am just choosing the direction as like this from right to left. Let us denote this current \( i \) which is flowing from this right point of \( R_f \) to the left, so it will be not entering the op-amp. We know that this is an ideal op-amp analysis. Current entering into op-amp is zero, so it is bound to flow in this direction to the ground. Current direction is this way. Same current will flow through \( R_1 \) to ground because there is no current into the op-amp.
We can use again Kirchoff’s voltage law. What will be Kirchoff’s voltage law in this loop? Starting from $V_0$, $V_0$ minus $i$ into $R_f$ minus $i$ into $R_1$ equal to zero because this current entering into the op-amp is zero and from that, the expression for $V_0$ is given as $V_0$ equal to $i_1 R_1$ plus $R_f$ because from this expression I get $V_0$ minus $i$ into $R_1$ plus $R_f$, that $i$ taken common, equal to zero. We can write down the expression for $i$ first. $i$ is equal to $V_0$ by $R_1$ plus $R_f$. The current expression we have found out. Again we can come from $V_0$ to the ground in this way also. That is another closed loop and that loop if we consider and apply Kirchoff’s voltage law what we will get? $V_0$ minus $V_1$ equal to zero; it will not lead to a good expression, but we will use $V_0$ minus $i$ into $R_f$ minus $V_1$ is equal to zero. That means we will come from $V_0$. We will come through $R_f$ through $V_1$ to ground.

If we start from $V_0$, it will be $V_0$ minus this drop $i$ into $R_f$ and I am coming down. There is no voltage drop in between these portions because these two voltages are same; so minus $V_1$ equal to zero. That is this expression $V_0$ minus $i$ into $R_f$ minus $V_1$ equal to zero. We have two expressions; one is for this loop and one for this loop. The second loop we are getting the expression for $V_0$. Now replacing $i$ from the earlier expression $i$ is equal to $V_0$ by $R_1$ plus $R_f$ into $R_f$ minus $V_1$ is equal to zero. Taking $V_0$ common, $1$ minus $R_f$ by $1$ plus $R_1$ plus $R_f$ equal to $V_1$ transferring $V_1$ to the right side.
From this expression basically we get \( V_0 \). If this expression is simplified it will be \( R_1 \) by \( R_1 + R_f \) equal to \( V_1 \). \( V_0 \) equal to, cross multiplying we get, \( V_1 \) into \( R_1 + R_f \) by \( R_1 \). Finally we get this expression of \( V_0 \) equal to \( V_1 \) into 1 plus \( R_f \) by \( R_1 \). So this is also a very important example of op-amp application which is non-inverting amplifier. Its output is given by the input voltage into 1 plus \( R_f \) by \( R_1 \). This is for non-inverting; earlier minus \( R_f \) by \( R_1 \) was the gain for inverting op-amp and here the gain is 1 plus \( R_f \) by \( R_1 \).
Now let us consider another application. What is this circuit? If you observe closely, output voltage we will have to find out. What is this voltage? This voltage is nothing but the voltage here. Again this voltage is equal to the voltage here and this voltage is nothing but \( V_1 \), which is the applied voltage. So \( V_0 \) is equal to nothing but \( V_1 \) and that is generally known as a voltage follower. That means whatever input voltage you are applying, output voltage is the same. Only it is used as a buffer; that means whatever input voltage we are giving, we are getting the same at the output. That is why it is called voltage follower where output and input voltages are same.

Now let us consider another application where you can sum up the input voltages. Suppose we are having a number of input voltages given to the op-amp; \( V_1, V_2, V_3 \), etc. The output voltage now can sum up the input voltages \( V_1, V_2, V_3 \) in a scaled manner. That means a scaling will be there in terms of the resistances; not that \( V_0 \) equal to simply \( V_1 + V_2 + V_3 \) but it will be scaled with the resistances being applied. By properly choosing the resistances we can make it as an exact summer also.

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In this application, we are applying three voltages. For example I am taking only three voltages. We can extend it to a number of voltages in the same way; that is the same principle is followed. Here $V_1$, $V_2$ and $V_3$ are the voltages being applied at the input having resistances $R_1$, $R_2$ and $R_3$ and the feedback resistance is $R_f$. The plus or non-inverting terminal is grounded. We have to find out what will be the output voltage? In these types of circuits it is always better to follow from the first principle and name the currents flowing. If we name the currents flowing through the three resistances as $i_1$, $i_2$ and $i_3$ and the current in this $R_f$ is say $i$, then what is $i$? If the current is here say $i$, the currents flowing in the three resistances we are naming as, say $i_1$, $i_2$, $i_3$. As no current enters into the op-amp, $i$ must be equal to $i_1$ plus $i_2$ plus $i_3$, the sum of all the currents that must flow through this $R_f$ only because they cannot go and enter into the op-amp. That principle we will be applying. Let us name the currents $i_1$, $i_2$, $i_3$ which flows in the three resistances $R_1$, $R_2$ and $R_3$ and the current which is flowing in $R_f$, let us name it as $i$. The principle is that there is no current into the op-amp. So, $i_1$ plus $i_2$ plus $i_3$ is equal to $i$.

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We will find out what is $i_1$, $i_2$ and $i_3$, individual currents. What is $i$? If we look into the
current $i$, applying Kirchoff’s voltage law start from ground; zero minus what is this $i$?
The drop between these two is zero minus $V_0$ by $R_f$ is equal to $i$. The potential drop
between these two points that is across the resistance is zero minus $V_0$ because this point
is nothing but this point and this point is nothing but this point which is grounded. So this
voltage is zero. So zero minus $V_0$ minus $R_f$ that is $i$ and what is $i_1$? $V_1$ minus zero by $R_1$
because this point is zero; so $V_1$ minus zero or $V_1$ by $R_1$. What is $i_2$? Similarly $V_2$ by $R_2$.
What is $i_3$? $V_3$ by $R_3$. So we get to the right side $V_1$ by $R_1$ plus $V_2$ by $R_2$ plus $V_3$ by $R_3$.
Simple manipulations we will do, so minus $V_0$ by $R_f$ equal to $V_1$ by $R_1$ plus $V_2$ by $R_2$ plus
$V_3$ by $R_3$.

What is $V_0$? Cross multiplying with minus $R_f$ into this whole term, so finally we get $V_0$
equal to minus $R_f$ by $R_1$ into $V_1$ plus $R_f$ by $R_2$ into $V_2$ plus $R_f$ by $R_3$ into $V_3$; that is the
expression for output voltage. It is like a summer. It is like summing of all the voltages
$V_1$, $V_2$, $V_3$. There is a minus sign in front because we are connecting at the inverting
terminals, all these voltages. If we look here all the voltages are connected into the
inverting terminal. So the minus sign will come as this will be 180 degree out of phase.
But if we look into the bracketed expression it is like a summer $V_1$, $V_2$ and $V_3$ are
summed up with a scaled down version, by scaling by the resistances; means $V_1$ will be
multiplied by this ratio of $R_f$ by $R_1$. But properly choosing $R_f$, $R_1$, $R_2$ and $R_3$, now we can get whatever sum we desire. For example I want to get $V_0$ is equal to say minus $3V_1$ plus $5V_2$ plus $6V_3$, etc; some example if we take then we can determine what should be the ratio between $R_f$ and $R_1$, $R_f$ and $R_2$ and $R_f$ and $R_3$ and properly choosing these resistances, I can get the required sum. That is why it is called the summer. So in analog computation, when summing up is required, MS, in an analog computer where you are summing up the analog voltages, that is done by this type of circuits.

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Another application is an integrated circuit. We want to integrate a voltage. Integration, differentiation, these are the mathematical operations we want to perform by using analog computation. Here it is analog amplifier. We are not considering digital, we are considering an op-amp where the whole operation is analog. So in analog computation, the summer, subtractor, integrator, differentiator, etc can be performed using op-amp and IC 741 is an IC integrated circuit where you get this op-amp. If we want to perform an integration operation on a signal; suppose I want to integrate $V_1$ with respect to time that I can do with a circuit like this where there will be a resistance and capacitance and they are connected to the op-amp. If we want to find out $V_0$ given that the voltage $V_1$ is
applied at the inverting terminal and a resistance is there, which is R and this capacitance is there across this op-amp which is C. What will be \( V_0 \)?

In order to know \( V_0 \), now we will use similar analysis, the way which we followed earlier. We will name the current. Suppose input current \( i \), flowing from \( V_1 \) cannot enter into op-amp; so the same \( i \) will flow out though C.

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Now if we apply Kirchhoff’s voltage law, there will be two loops which we can consider. First loop if we consider, this \( V_1 \) minus \( i \) into \( R \) is equal to zero; that is one equation we get and from that we get the value of \( i \) and that is equal to \( V_1 \) by \( R \). Again if we consider the other loop which is flowing from this ground that will be zero minus \( i \) into \( R \) minus \( V_0 \); \( V_1 \) minus \( iR \) is equal to zero and the other is from here if we consider zero minus the voltage across this capacitance \( 1 \) by \( C \) integration \( idt \) zero minus \( 1 \) by \( C \) integration \( idt \) is the voltage across this capacitor.

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So the voltage drop across this capacitor is \(1 \text{ by } C \int dt\). If I proceed from ground and apply Kirchoff’s voltage law, zero minus \(1 \text{ by } C \int dt\) minus \(V_0\) equal to zero. This is the equation. From here we get \(V_0\) which is equal to minus \(1 \text{ by } C \int dt\). Now I can replace \(i\) which is equal to \(V_1\) by \(R\). Doing that we get minus \(1 \text{ by } C \int V_1 \text{ by } R \int dt\). Finally what is \(V_0\)?

\(V_0\) equal to what we get? \(R\) can be taken out, \(1 \text{ by } R\) can be taken out, it is outside the integral. We can take out, it is the constant. So minus \(1 \text{ by } R_C \int V_1 \int dt\). That is the value of \(V_0\). It is out of phase. Minus sign is there that means it will be out of phase with \(V_1\) signal. But if we consider this expression of \(V_0\), it is nothing but integration of \(V_1\). Basically we are getting an output which is integrating \(V_1\). This \(1 \text{ by } R_C\) term is there, it will be scaled down by \(1 \text{ by } R_C\). It is integrating the input voltage so that is the operation being performed by the connection of this op-amp in such a way with \(R\) and \(C\) connected in this fashion that we finally obtain the integration operation.

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Similarly we can get a differentiation operation. If we want to differentiate a voltage $V_1$, then the positions of R and C will be just interchanged. Here it was C here and R here. We will interchange C and R; C will come to the input and R will be across this op-amp.

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What will be the output voltage? Proceeding similarly, we will be get by applying the Kirchoff’s voltage law again. What is this Kirchoff’s voltage law being applied? That is $V_1$ minus the drop across this capacitance. That is this voltage drop across this capacitance.
capacitance is \( V_1 \) by \( C \) integration idt. So \( V_1 \) minus 1 by \( C \) integration idt, you come down to ground. That is \( V_1 \) minus 1 by \( C \) integration idt equal to zero.

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This is one equation when you come to ground starting from here. Another is, you go in this direction. That will give you zero minus \( i \) into \( R \) minus \( V_0 \) equal to zero. So this is the other equation. Manipulating this equation a little, take the differentiation on both the sides; \( dV \) with respect to time, \( dV_1 \) by \( dt \) equal to 1 by \( C \). \( dt \) of integration idt means it will be simply free in the term \( i \). So it will be 1 by \( C \) into \( i \). What is \( i \)? \( C \) into \( dV_1 \) by \( dt \).

From the second equation, we get what is \( V_0 \)? Zero minus \( iR \); that is equal to, \( i \) can be replaced from here; \( C \) into \( dV_1 \) by \( dt \) into \( R \). So \( V \) is equal to minus \( RC \) \( dV_1 \) by \( dt \). If we look into this expression, this is nothing but it is differentiating the voltage \( V_1 \). We get a differentiation of the signal \( V_1 \) by this connection of this op-amp to the resistance and capacitance in this way and in the earlier case, we were getting a integration operation. These two are examples where the op-amp is being used in a circuit which will give you integration as well as differentiation. These mathematical operations can be performed by the op-amp.
Let us take another example, which will show how the subtraction operation can also be done. We have done addition, we have done integration and we have done differentiation with op-amp. Now substraction or difference between signals can be found out.

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Here the two signals being applied are \( V_1 \) and \( V_2 \), the resistances connected are \( R_1 \) and \( R_2 \). First we denote the node voltages which are at the nodes, a and b. For our easy analysis, we will name the nodes as a and b that is the inverting and non-inverting terminal node voltages are denoted by \( V_3 \) because we know \( V_3 \) the voltage at node a is equal to voltage at node b. Both the voltages are equal. So let us name it as \( V_3 \). This \( V_3 \), we are introducing just to easily analyze the circuit. Because we have the nodes we can apply Kirchoff’s current law.
What is Kirchoff’s current law? It is the algebraic sum of currents at a node equal to zero. Algebraic sum means plus and minus depending upon what direction it will be. We will consider node a and we will apply Kirchoff’s current law. The Kirchoff’s current law being applied at node a, the sum of incoming current is equal to the sum of outgoing currents. So $V_1$ minus $V_3$ by $R_1$ that is incoming, this direction and that $V_1$ minus $V_3$ by $R_1$ must be equal to the, this is the incoming, the outgoing which is $V_3$ minus $V_0$ by $R_2$. So that is equal to $V_3$ minus $V_0$ by $R_2$. This is applying Kirchoff’s current law at node a. Applying Kirchoff’s current law at node b, $V_2$ minus $V_3$ by $R_1$ is the incoming current. Which one is the outgoing current? There cannot be current going into the op-amp because this is ideal op-amp consideration. The current which will be coming out from terminal b is equal to $V_3$ minus zero by $R_2$ because this part is grounded; that is equal to $V_3$ minus zero by $R_2$ means $V_3$ by $R_2$. So we get $V_2$ minus $V_3$ by $R_1$ is equal to $V_3$ by $R_2$.

These two are the key equations, which will be rearranged. Rearranging equation 1, transferring this term to the other side, take $V_3$ common. It will be 1 by $R_1$ plus 1 by $R_2$ and then minus $V_1$ by $R_1$ that is equal to $V_0$ by $R_2$. I am transferring this to the right side and the term having $V_0$ by $R_2$ to the left side. This equation 3 is the rearranged form of equation 1. Similarly rearranging equation 2 I get, taking common $V_3$, $V_3$ into 1 by $R_1$
plus 1 by R₂ minus V₂ by R₁ equal to zero. Now if we look into the two equations 3 and 4, we can very easily get rid of the term V₃ just by subtracting 4 from 3. If we subtract the equation 4 from equation 3, what will it give us?

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These two terms will go; it will be positive. So we get V₂ by R₁ positive minus V₁ by R₁ equal to V₀ by R₂. From this we get what is V₀? Cross multiplying by R₂ we get V₀ equal to R₂ by R₁ into V₂ by V₁. If you now observe this expression what we get is that we are basically having a difference between the two voltages V₁ and V₂. So V₁ and V₂ were the voltages which were applied. V₂ is the voltage which was applied to the non-inverting terminal, V₁ was the voltage applied to the inverting terminal. V₂ minus V₁ this algebraic difference is obtained at the output. It is multiplied by a factor R₂ by R₁ where R₂ and R₁ are the resistances which were connected in the circuit. One thing that is clear is that we can get this difference operation also from a circuit connecting the op-amp properly with resistances. The factor R₂ by R₁ can be chosen; if we choose the values of R₁ and R₂ appropriately, then we can get the factor you want to multiply that difference or even if you want to make R₂ and R₁ equal that will cancel and we will get only the absolute difference which is V₂ minus V₁ if R₂ equal to R₁. In this way we have seen that we can get a difference operation between two voltages also done.
Now let us do one example to sum up all the discussions today.

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In this circuit you have to find the ratio between $V_0$ and $V_1$ or $V_1$ that is the input voltage $V_1$. What will be the value of $V_0$ by $V_1$? Given that the resistances are connected in such a fashion, this is $R_1$ which is connected to $V_1$, this is $R$, $R_2$, this is also $R$. To analyze such a type of circuit the easy way which we have been following we will still adopt, the same way of naming the currents. Input current let us name by $i$. If this current is $I$, there cannot be any current into the op-amp following ideal op-amp consideration.
The current has to go in this direction, this current i has to go in this direction. So i will be following in the direction through R at this point. I am naming this voltage at this point as V. This voltage I am naming for easy analysis as V. So at this point the current will be divided into two parts. One will go to the R to ground downwards which is say i_1, I am naming and the other is i_0 which is flowing through R_2. So there are divisions of current at this point.

(Solution)

\[ i_0 = i - i_1 = \frac{v}{R_1} - \frac{v}{R} = \frac{v}{R_1} - \left( \frac{-iR}{R} \right) \]

\[ = \frac{v}{R_1} + i = \frac{v}{R_1} + \frac{v}{R_1} = 2\frac{v}{R_1} \]

\[ v_0 = v - \frac{i R_2}{R} = -i R - 2v R_2 R_1 \]

\[ = -\frac{v}{R/R_1} - 2v R_2 / R_1 \]

or, \[ v_0 = -v(R + 2R_2)/R_1 \]

Hence, \[ \frac{v_0}{v_1} = \frac{(R + 2R_2)/R_1}{R_1} \]

\[ R = \frac{R_1}{R_1} = R_2 \]
Applying the Kirchoff’s current law or voltage law which ever is suiting, we will now know what is $V_0$? Following in that way now what is the current $i_0$? We can write down that $i_0$ current is equal to $i$ minus $i_1$. That is current division is taking place, so let us find out what is $i_0$? $I_0$ is equal to $i$ minus $i_1$. What is $i$ and what is $i_1$. From here I can see what is $i$? $i$ is nothing but $V_1$ by $R_1$; so $V_1$ by $R_1$ is $i$ because at this end, voltage is zero. So $V_1$ minus zero by $R_1$ is $i$. What is $i_1$? $V$ minus zero by $R$ or $V$ by $R$. Replacing this $V_1$ by $R_1$, $V_1$ input voltage let us take it as $V_i$. I am naming it as $V_i$ because this analysis I am doing with $V_i$. So $V_i$ by $R_1$ minus $V$ by $R$ that is equal to $i_0$ and that is equal to $V_i$ by $R_1$ minus ..

What is $V$? If we see what is $V$, $V$ can be found out if we start from zero. Zero minus $i$ into $R$ is equal to $V$. Zero minus $i$ into $R$ is equal to $V$ because that is evident from Kirchoff’s voltage law being applied like this from this terminal; zero minus $i$ into $R$ is equal to $V$. So $V$ is nothing but minus $iR$. Replacing this $V$ by minus $iR$ divided by $R$ is already there what we get is equal to $V_1$ by $R_1$ plus, minus, minus plus, this $R$, this $R$ cancel. So $V_1$ by $V_i$ by $R_1$ plus $i$ we get. Again we can write that equal to $V_1$ by $R_1$ plus, this $i$ is nothing but $V_i$ by $R_1$; we have already seen that. So replacing this $i_1$ by $V_i$ by $R_1$ what we get? This is $2$ times $V_i$ by $R_1$ is the expression for $i_0$. What is $V_0$? We can write down $V_0$ as $V$ minus $i_0$ into $R_2$. $V$ voltage here minus this drop equal to $V_0$. So $V_0$ equal to $V$ minus $i_0$ into $R_2$. What is $V$? Minus $iR$ we are replacing. What is $i_0$? We have found out $2$ times $V_i$ into $R_2$ by $R_1$. This $i_0$, I am replacing by $2$ times $V_i$ by $R_1$ into $R_2$. Again $i$ is nothing but $V_i$ by $R_1$ into $R$ minus $2$ times $V_i$ $R_2$ by $R_1$. Simplifying this expression taking common minus $V_i$ by $R_1$ we get $R$ plus $2$ times $R_2$. If we find the ratio between $V_0$ and $V_i$ that is required, $V_0$ by $V_i$ is required we get $V_0$ by $V_i$ is equal to minus $R$ plus $2$ $R_2$ divided by $R_1$. This expression is in terms of the resistances which are connected in the circuit and we have got that the ratio between the output and input voltage equal to minus into the resistances $R$ plus $2R_2$ by $R_1$. These resistances which are connected in the circuit we are taking these resistance values as this is $R$, this is $R$, this is $R_1$ and this is $R_2$.

Now if we choose particular values of these resistances, actual values which we will be using, it will give say $1k$, $2k$, etc, whatever values we apply. Then your voltage ratio will be in terms of those values because you can see here if all are equal suppose $R$ is equal to
R₁ equal to R₂, so what we will get? Minus R plus 2R divided by R and that is equal to minus 3R by R, so -3. So V₀ by Vᵢ becomes -3. That means output voltage will be -3 times the input voltage. It will be out of phase with the input voltage and it will be amplified 3 times. If we have a signal which is 1 millivolt, we will get 3 millivolt but minus that is out of phase with the input voltage. So properly choosing these resistance values, whatever, we are connecting in this circuit; this is the typical circuit, where the ratio between these two voltages that is output and input voltage will be determined by the values of the resistances which you are connecting. Properly choosing these resistance values in terms of what you want to get, how much ratio between the two voltages you want to get in that way you will be choosing these resistances.

For example just I have shown that if we want to get output voltage to be 3 times the input voltage with a minus sign because we are connecting the voltage at the inverting terminals. We will have to choose all resistances equal that will give you 3 times. In this way you can choose the resistance values and you can find whatever required output voltage you want in comparison with the input, how many times you wanted to be greater.

In today’s discussion we have seen basically the ideal op-amp characteristics or properties of an ideal op-amp that it should have infinite input resistance and voltage gain should be also infinite and we have used these properties extensively in finding out the output voltages for different various types of applications. Practical op-amp applications we have seen starting from inverting, non-inverting op-amp, up to summing or difference amplifiers, also integrator or differentiator etc. In all these practical op-amp applications we have used the ideal op-amp analysis and we have used the principle of an ideal op-amp that input current entering into an op-amp is zero and the gain is infinite that is why the two voltages at the input terminals, that is negative and positive terminals, these two voltages are equal. These analyses we have extensively used. So the analog computation can be performed using an op-amp in a proper combination of resistances as well as capacitors and we can get the different analog computation starting from summing,
difference up to integration, differentiation, etc. This is the versatility of op-amp which gives us analog computation.