now we will now continue our discussion on the phase locked loop that we have started last time (refer slide time: 01:08)

to recap we had looked at the steady state operation of the phase locked loop that is assuming that the loop is in lock we had argued that under certain condition the instantaneous phase of the vco output of the loop would follow the incoming phase right will track the incoming phase in the process the derivative of the input phase d phi by dt and the derivative of the vco phase would be near would be nearly the same right and therefore the vco input voltage could be considered to be the replica of the modulating signal that we want it to get from the demodulator right

so that was one part of the discussion that we had to demonstrate that in the steady state the if you if you tap the vco if you if you tap the phase locked loop at the input to the vco or after the loop filter right after the amplifier then that output can be considered to be a demodulated output for the fm signal or the angle modulated signal that particularly for the fm signal that is under consideration right that was the first part

the second part was you try to figure out how the locking actually occurs right

now in this case we have to go through a non linear model of the loop right because we we found that the phase locked loop actually is a non linear system right it is in fact we obtain a differential equation description for the operation of the phase locked loop right
in particular we arrived at this non linear differential equation which specifies the instantaneous phase error between incoming phase and the VCO output phase right (refer slide time: 03:02)

so we found that this phase error follows this non linear differential equation right and you try to look at the operation of the phase locked loop by considering what does this differential equation imply in terms of its operation right and to do that we took request to a graphical picture graphical the picture has this differential equation through the so called phase plane plot right

in the phase plane plot basically we plot d psi by dt versus psi psi of t right so this obviously becomes the sinusoidal function with the peak occurring when sine of psi of t is either plus one or minus one right

so when it is plus one this this gives the delta omega minus kt that is a negative peak whether it is minus one this gives you the peak value as delta omega plus k of t which is the positive peak and at psi equal to zero this point intersects the d psi by dt ((axis)) ((04:24)) delta omega right

because at that point psi is zero so d psi by dt is equal to delta omega so this was a phase plane plot and then we discussed that this phase plane plot essentially gives you the trajectory of the operating point of the phase locked loop right

in the sense for example if initially you are somewhere here on the trajectory initially assuming that the steady state phase error was zero somehow initially when the we considered a case when all this discussion we did in the context of the step input in the frequency of the incoming signal right as if your moderating signal was a unit step function right of course we we can we cant do it in a general we can’t plot a phase plane plot for the general signal you have to do it for a specific signal and the specific signal we considered was a unit sub function right (refer slide time: 04:45)
so for this case assuming that initially the loop was in locked at the carrier frequency we found that the loop will eventually the if you move the operating point will move on this trajectory to the right right

and the second important point is the motion on this trajectory will be to the right if you are in the upper half plane of the trajectory the motion will be to the left if you are in the lower half plane of the trajectory and based on this arguments we concluded that this point a is a stable operating point of the loop which means that after the frequency step has been applied the loop will eventually try to come to this condition right

because at this point d psi by dt becomes zero that means there is no further movement in the operating point because there is no derivative there is no slope psi as the function time becomes the constant value right and the constant value is this value whatever is the value here this is psi axis so whatever is the value of psi at this point is the steady state value of the k phase error right [student] they will they will they will be many such points but they all periodically placed right connect such point will be at this pha this value plus two pi right

so in a in as much as phase is anywhere specified modulo two pi it doesn’t any matter which point you lock on to it hardly matters

phase error we cannot distinguish between the phase error which is of one degree or three sixty one degrees its a really the same thing right so that doesn’t really matter ok

so we now proceed further and just do a further little bit of discussion on the phase on this what we get from this phase plane plot

let's call the steady state phase error of lets denote it by psi sub ss right and of course we said the steady state frequency error would be zero sorry that’s the frequency error can you tell me what is the value of psi sub ss psi ss is it possible to see from this equation or this plot if you look at this equation once again at the ope at the stable po point what is the value of d psi by dt that is equal to zero right
so put that equal to zero you can solve for the corresponding value of psi right that will be sine inverse of delta omega by kt

so this will be sine inverse of delta omega by k sub t right these point now what does it show it shows some very important facts that if you want the steady state phase error to be small what should happen kt’s will be large for a given frequency step at the input if you want the steady state phase error to be small this argument should be small which will happen so for a small value of psi ss kt should be large (refer slide time: 11:49)

and what is k sub t if you remember it is the loop gain right isn’t it so basically we are saying that we require a large value for the loop gain

now we require a large value for the loop gain not only for this purpose but for another very important reason right

let us see all this discussion was nicely concluded and we we could demonstrate the phase plane plot that indeed the loop will lock right in as much as the phase error thus comes to a small value it doesn’t become zero of course if you want it to ma make it zero if you want to make this steady state phase error equal to zero you should have a loop gain of infinity right

you must have a very very large value of loop gain so the larger the value of the loop gain the smaller you can make this steady state phase error right

but nevertheless whether large or small loop gain the important point was that for it to be be come to the stable point it should have a stable point isn’t it

now if there is situation where a stable point may not be there at all [student] then delta minus kt is a positive number right

so as it in that case this phase plane plot this curve will not intersect this i axis at all and you will always be in the positive half plane the loop will the trajectory of a operating
point will always keep on moving the operating point will not be able to find a point at which it can stabilize

you keep on attempting to lock but we will never really be able to lock right there is very interesting way of looking at it right

so the second important thing to note is that for the lock to occur so this was lets say the first point the value of the steady state phase error

for the lock to occur the phase plane trajectory or phase plane curve should intersect this i axis right

and in order for that to happen what do you need you need to make sure that delta omega minus k sub t is less than zero right

the negative peak of this curve should be negative i mean the lower peak of this key should be negative which implies that the delta omega should be less than the loop gain

so the loop gain plays a very important role in the operation of this system right

number one the large value of the loop gain will imply the small value of the pha steady state phase error

the large value of the loop gain will also imply a large value of the what is called locked range of the loop right

so loop gain in a way besides what is called the locked range or locking range of the pll because locking range of the pll is the maximum frequency step that you can employ that you can have at the input and still the loop is able to produce a steady state situation that a frequency error is zero that is the vco exactly locks to the input signal frequency right

so locking range of pll is equal to k sub t and large lock range requires that we choose a large value of kt ok

let me summarize this discussion by showing you a plotting a set of phase plane curves but different values of k sub t sorry for for different values of delta omega or delta f right

lets for this discussion assume that your loop gain is has some fixed value lets arbitrarily chosen the fixed value to be two pi into fifty right

so lets consider the phase locked loop in which the loop gain has this value right

so let me now plot the phase plane plots a di for a different sets for different sets of delta omega right (refer slide time: 16:03)
so when i choose lets say delta f equal to fifty five hertz right that you will say apply a frequency step of that frequency step of fifty five hertz of the input

initially the loop is in lock and you apply a frequency step of fifty five hertz it means delta omega is two pi into fifty five right

so this condition is not satisfied which means the loop will not the the directive will not intersect the psi axis you will get some thing this right sorry you will get something like that it will not intersect the psi axis at all

suppose you make it forty eight hertz it will almost just intersect now right but intersect it here

so we will now be in the situation like that etc right so a stable operating point will exist but the corresponding steady state error would be quite large right if you make it half of this so this is fifty five hertz this is forty eight if i make it half of this as you can see the steady state phase error will come down accordingly if i have it further it will come down further right

so this is the sequence of curves that i have plotted fo for delta f equal to fifty five hertz forty eight hertz then twenty four hertz and twelve hertz ok

so this picture says it all it demonstrates very clearly that as we increase the loop gain we get better and better performance from the phase locked loop any questions here [student] sorry [student] no [student] the same thing will apply you thing about it the frequency step in either direction would imply the same thing ok

ya in a way you are saying ok if i agree with you what what you are saying is precisely the same thing what happened thank you ya you can make it del delta omega model because positive of frequency negative only the roles will get into exchange ok

so that also means something else that means even in the steady state i cannot always use a linear approximation that i discussed earlier do you agree that because a linear approximation to be valid what is a assumption that you made that psi of t is very small
but suppose your psi your loop gain is not very large and delta omega is quite large right you could have a very large value of the steady state phase error and therefore for that value of the steady state phase error i cannot assume sine of psi of t is equal to psi of t right

so the linear approximation that we considered would not be available for use even in the steady state if your steady state phase error is not very small right

so it is therefore imperative that if you want to use linear approximation in the steady state at this ((17:57)) [noise] ok

and it is it is useful to look at that small signal appro small phase error approximation or linear approximation to study the steady state behavior of the loop right at least approximate behavior if not exact (refer slide time:22:37)

so when psi ss is small we can use the linear approximation can be used for analysis what is it mean if remember your final equation from which we derived the differential equation first that theta of t was equal to k sub t integral of theta of t is a instantaneous phase of the vco output right this all this was equal to k sub t times there it was sine of phi alpha minus theta alpha right

if you are going to use a linear approximation that sine can be removed and we can simply write it as phi of alpha minus theta of alpha d alpha right and if you want to convert this into the corresponding differential equation this will becomes d theta by dt plus kt theta t so instead of sine theta t you have kt theta t is equal to instead of kt sine psi t right you can write this kt phi of t this is a linear differential equation

so when you when the linear approximation is valid the loop is now describe at or near the steady state by a linear differential equation right and when that is so it is possible to study it much more conveniently because we know how to study this linear differential equation lets say by laplase transforms in fact we can obtain a transfer function model of the loop right
for example i could define a loop transfer function sometimes also called the close loop transfer function with as a ratio of the laplace transform the output and the input and what is the output here output is theta of t the final phase of the vco right and the input is phi of t the incoming phase function right

so i define the loop transfer function h of s as theta of s upon phi of s definition of any lapla any transfer function is laplace transform of the output variable laplace transform of the input right that’s what we are doing

we are considering the vco output as vco output phase as the output here may be the laplace transform of that the input is the incoming phase function which is phi of t laplace transform of that is phi of s ok

so what is what will be the value of the transfer function here you can derive it from here by just taking the laplace transform of both the sides can you tell me what will be the value of theta s upon phi s from this differential equation it will be simply k sub t upon s plus k sub t right

and the corresponding ht the corresponding impulse response we can write as k sub t e to the power minus k sub t into t ut right that shows that is unit step at the input there is not produce the unit step at the output in which you create goes through the exponential function sorry impulse function of the input not the unit step this is the impulse response

an impulse function at the input does not produce an impulse function in terms of the output but but some other impulse some response which is an exponential ((23:09)) response ok

this will ideally if you don’t want the system to do any distortion of any kind then this cond then this h of h of t should be equal to delta t itself right under what condition will this converts to delta t when kt it becomes very large

so ht will converge to delta t or will tend towards delta t as kt tends to infinity right and when that happens right when that happens your theta of t would be the same thing as phi of t right there will be no phase error for large loop gain so no much of how you look at it needs the same result ok

before we proceed further suppose i ask you the question that i want to use the pll for demodulating phase modulation signals phase modulated signals what will i do how should i do that demodulation of pm signals what’s the answer you should integrate the output of the demodulator of fm demodulator right

so just take the integral at integral of e sub t so integrate e sub vt that will produce a required demodulator lets do for any you know you have the fm demodulator output and if you want the demodulator phase modulated signal you just have to integrate the output of the fm demodulator right standard result (refer slide time: 29:02)
some final remarks before we go on to some other discussion of the pll yes is there a question ok

so what we notice from this discussion is that the large loop gain is crucial right large loop gain incidentally if you look at this transfer function i can think of suppose you think of this transfer function purely as a filter right what is a bandwidth of this filter lets say the three db bandwidth

now looking at the loop func transfer function as the filter right you have transfer function kt upon s plus kt what is a three db point of this transfer function it’s a low pass function you can see that isn’t it as omega increases the function the value will come down right it will attenuate the signal more and more what is that three db point s is equal to kt right

so you can think of kt as the three db bandwidth of the loop right or k sub t also so k sub t has a lot of significance k sub t is the lock range k sub t besides the steady state value of the phase error k sub t can also be considered to the to be what is called the loop bandwidth right

so k sub t can also be construct with in this particular case the loop bandwidth right so what we are saying is that if we want a large if you want a good performance from the phase locked loop in terms of steady state phase error in terms of lock range and consequently in terms of demodulation of fm signal right

you would require either a large loop gain or equivalently a large loop bandwidth right

now that is usually so if you if you want to the pll to work properly as a fm demodulator again you want large value of k sub t these are the conclusions we are instinct now large loop gain is not always possible right its very difficult it also has some problems associated with it right which will not go into right now but its usually not easy to realize a very large loop gain
so you need to think of improving the performance when we don’t have sufficiently loop gain right and one way of improving the performance is you know that this loop that this analysis that we have done is for a very specific situation when we have removed the loop filter right

so one of the advantages of using the loop filter including the loop filters is that we can relax on this condition that we have on the loop gain right so loop presence of loop filters helps

only thing is the moment we have loop filters our analysis becomes much more complex isn’t it because i can no longer write that differential equation so easily have to also consider the how the filter effects the differential equation right

so far the phase detector output except for the multiplication by the loop gain was directly going into the vco input right

so it is very easy to relate the vco phase output to each input right it was very easy through that integral or through the differential equation the moment you have an addition of filter coming between these two you have to also see how that affects the entire description in terms of either differential equation or integral equation [coughing] its not a cannot be done it just becomes little more complicated at the moment we will not going to that but we will have a very deep discussion of the at least one more loop where the loop filter

now the second remark is look at the value of k sub t that is given by i don’t remember the exact value it is half ac ab k sub d anything else mu there are so many factors which determine the value of the loop gain what is it mean if you first of all input signal amplitude itself has a effect on the loop gain right (refer slide time:32:24)

now other thing of course more or less constant of the pll itself av is associated with the vco k sub d is associated with the phase detector mu is the gain that you have incorporated in the phase locked loop right
so these three are constants of the loop but a sub c is a constant associated with a input signal that’s not a very nice thing to have you don’t want a loop design to be dependent on the amplitude of the input signal because if that is so the ampli input signal has the weak amplitude the loop gain goes down input signal has a stronger amplitude the loop gain goes up right lets not very desi desire about

so dependence of because otherwise what we have to do is we have to design the loop for operating to a certain signal amplitude right and the signal amplitude changes if you have value of k sub t changes and then you all your design is gone right so dependence of design on input amplitude not desirable can you suggest a way of removing this dependence [student] and show that the input signal always has a constant amplitude right input fm signal always has a constant amplitude and we know one one method of doing that the bank pass limiter right

so we can precede the pll by a bank pass limiter good that’s a very good answer ok [student] good question i think i forgot to mention that

the question let me repeat the question for this complete discussion i have taken the input signal to be a cosine function and the phase modulator fun the vco output to be a sine function right why did i do that when you see the [student] that’s right because i am how am i real it depends on how you realize your phase detector i am realizing the phase detector by multiplying the two signals and low pass filter in the output right when i multiply cosine with sine i will get a the difference the difference component will be sine

if on the other hand multiply cosine with cosine the difference frequency func function the difference function will be a cosine function the cosine function will not be sensitive to the sine of the phase error right it only depend it will only dep dep decide the magnitude of phase error and you cannot track the phase locked loop will not be able to work properly because it must not only know what is the phase error so that the phase error can be driven to zero it must also know whether the phase error is positive or negative right you want to be sensitive to the phase error right

so it is important that this wo its also automatically works like that right so in a locked condition it is implied that if the input signal is cosine the output signal will be sine that is there will be pi by two phase shift between the input signal and the demodulated signal right

so the input sign carrier is cosine to phi of ct the vco output will be sine to phi of ct right this will automatically happen because you have close loop and the close loop will work like that ok that was the good question i forgot to mention about that earlier ok

now what can we do about the removal of these disadvantages of the first loop the first order loop right this loop that we discuss is called a first order loop right

so loop the pll without the loop filter now can you also see why it is called a first order loop [student] that is one way of looking at it and another consequence of that is through
the transfer function that we had the loop transfer function \( k_t \) upon \( s + k_t \) is the first order transfer function right

so that is why we call it a first order loop right it has the disadvantages that we already seen there are two basic disadvantages which lead to non optimum performance one is a limited lock range and the second is non zero steady state phase error right these are the two disadvantages that we are discussed can we do something about it as i mentioned we can try to remove both these disadvantages by using a loop filter of an appropriate kind

to illustrate that can be done that in deed happens let us consider a loop in which we choose a loop filter to the transfer function like this

this is the transfer function not of the loop but of the filter in the loop that we are going to put right which we are omitted in this discussion right (refer slide time:39:08)

this is as you can see essentially an integrator right the second part \( a / s \) is an integrator right the first part is one which allows your input signal to just go out right the such an integrator is called a leaky integrator right because it allows a input signal to leak through to the output as well as the component will come after integration right

so this essentially what is called a leaky integrator so basically your linear model lets consider the linear model for the simpli simplicity of discussion because the non linear model will become very complex now right

the phase plane plot the the differential equation will be second order differential equation now right because if if one order differential equation is contributed by the loop filter and one order the differential equation contributed by the relationship between the vco output and the vco input right because there is an integral relationship there right

so this will become a second order differential equation and the phase plane plot will be much more complex now right we will not go in to all that discussion
we will consider the steady state operation right i will at least through prove through steady state argument that you need it is now possible to get both these things are you looking for right

so we we look at that [students] no i think it need not like to have any phase error after after the locking has occurred as long as the phase error is there we we should know whether it is positive or negative so that loop can operate itself to drive the phase error to zero

the ultimate goal of the phase lock loop and that is why it is so called is to drive the phase error to zero right in whichever direction is the current phase error may be [student] i thought i already answer that question if you still have the doubt we will discuss it separately right ok

let me return to this so let me consider the small error model or the linear model for this case right

so what is the linear model for this case we have this i have removed the sinusoidal non linearity here because i am considering a linear model the input is phi of s the vco output is theta of s right this is the loop gain i am denoting it by a block and what we are now introduced additionally is a loop filter with transfer function f of s as given

the output of this is fed to the vco and the vco here will be represented by a transfer function of one by s its an integrated right so its transfer function is one by s and you are taking the demodulated output over here right

so what is the close loop transfer function here h of s equal to theta of s upon phi of s we need to find that out right if you do that so we need to find this to to be ((40:30)) proceed step by step lets write an expression for theta of s which is equal to can i write it in terms of this point here

suppose i go through the loop like this it is kt times fs times into one by s of the fully transform of the signal here and what is that phi of s minus theta of s

so basically it is k sub t into fs upon s of phi of s minus theta of s right ((41:13)) in the transfer function it’s a cascade of three functions

now you can solve for theta of s take this theta of s to the left hand side lets solve for that so if you do that and compute theta s upon phi of s its very easy to do that so i leave that as an exercise you can check this out and this becomes equal to kt times fs upon s plus kt times fs right and substitute for f of s as equal to s plus a upon s this becomes kt times s plus a upon s square plus k sub t s plus k sub ta ok so that’s what you have here

so it becomes a second order loop because your transfer function is a second order transfer function right (refer slide time:42:44)
so when you use a first order filter as a loop filter which is a first order filter the loop becomes the second order filter sec second order loop right

the loop transfer function has denominator degree which is two right ok

we are interested to look at the phase error so it will be interesting to look at psi of s the phase error the phase error is phi of s minus theta of s if about to consider the transfer function let us say between psi of s and in the input phi of s that will become one minus h of s if i divide both sides with phi of s right (refer slide time: 45:40)

and if i do that you can see that if i substitute for h of s from the expression that we just derived this expression here this will become s square upon the same denominator as before s square plus kt s kt as plus kt [student] kt [student] ok it will be kt s so i rewrite it s square by s square plus k sub ts plus k sub ta right ok

now from this can i say something about the steady state phase error from this transfer function is it possible to say something about the steady state phase error even without going through that non linear analysis there is a final value theorem that you know right the laplace transforms
what is the final value theorem suppose i want to find limit of psi of t st tends to infinity right this is limit of s tends to zero of s time psi of s of course to do that i must first consider some kind of input right i have to say something about phi of s

lets present the same kind of input that we are talking about earlier lets say that input is a frequency step right so if the input is a frequency step what can you say about phi of t the input frequency is a frequency step what happens to the phase will be integral of that right the phase will be integral of the phi d phi by dt d phi by dt has a frequency step right

so phi of t rather phi of s what can you say about phi of s so for a frequency step input the frequency step of delta omega right you can say that the phi of s is nothing but see frequency step step itself use that the laplace transform of this would be delta omega by s right (refer slide time:47:45)

laplace transform of the integral of that will be delta omega by h square so phi of s will become delta omega by h square are you with me on this all of you

the frequency step that is your d phi by dt is delta omega ut right that is what a frequency step of the input means right

so therefore phi of t would be integral of this the laplace transform of this is delta omega by s because laplase transform of ut is one by s right

laplase transform of phi of t will be another multiplication of one by s so that becomes delta by delta omega by h square

so now substituting that in the expression for psi s upon phi s i am taking phi s on the right hand side what do you get psi of s would be equal to one upon h square plus kt s plus kt a ok
now take the limit of $s\psi$ of $s$ as $s$ tends to zero what do you get it becomes zero right so it is implies that the steady state phase error is right because this is nothing but what you want limit $t$ tends to infinity of $\psi$ of $t$ so the steady state phase error is equal to zero

similarly you can argue i leave that as an exercise for you to complete that the steady state frequency error would be this in fact that’s trivially obvious from this because frequency error is going to be derivative of the phase error right so if this is zero that will also be zero ok

so a second order loop removes both the disadvantages of first order loop right however the only difficulty is the complexity of analysis this analysis that we have done is a very approximate linear analysis which is valid around the locking situation whether loop is nearly in the lock

so that the linear approximation can be assumed to be valid otherwise the proper non linear analysis which will of course lead you to the same conclusion right would actually show much more clearly how the locking actually occurs because you will have to be able to again see that how the track operating point varies as a function of lets say $\psi$ right

in this case you will find that the phase plane plot is not a very simple sinusoidal curve it’s a very complex curve and it does not it’s a it’s a sine its typically sinusoidal curve which gradually keeps on approaching the $\psi$ axis right and eventually needs or cuts the $\psi$ axis at the at a fair equal amount of distance after going to many many cycles rather than every cycle [noise]

in the first order loop the phase plane plot was the sinusoidal curve was intersecting the $\psi$ axis every cycle every two pi radians right that will not happen there right

so the actual non linear analysis will show all that that there is a lot of what is called cycles slipping taking taking place before the lock actually occurs but i am skipping all that analysis

we don’t have time for all that its a very detailed treatment of the phase lock loop which you can if you are interested learn by yourself it’s a very complex system to study because it’s a non linear system ok

so i think that is sufficient discussion for the phase locked loop but this is not the only kind of feedback demodulator we can have for frequency modulation right

there is another very interesting variant of phase locked loop which is often used in for demodulating fm signals more or less [noise] for the same purpose that the pll is used right which is called system with frequency compressive feedback which is very similar to a phase locked loop but there is there is some major differences
lets look at the block diagram you have an input signal \( x s x \) sub rt i have a multiplier i
don’t call it a phase detector any more i call it a multiplier i had the phase detector also as
as a multiplier but that was followed by a low pass filter right so that to make the phase
detector right

i will not follow it up with a low pass filter i will follow it up [noise] with the bank pass
filter that may be very surprising and because this this will not make sense if the carrier
frequency here and here are same isn’t it because if the carrier frequency here and here
are same then the difference component will always be proportional will always
be center around zero right

so i don’t bank pass filter serve will serve no purpose so it obviously implies that the vco
output are is not at the same carrier frequency as the input or in the lock condition also it
is at a different carrier frequency right here as in the phase locked loop the carrier
frequency at the vco or the lock conditions will be the same as that of the input signal
right

so therefore it is implied here that the vco works differently here now what will happen
then that you will have a signal here which is at a some finite carrier frequency which
will depend on the centre frequency of this bank pass filter and will depend on the
frequency of the vco here right (refer slide time: 53:57)

this i will follow it up with discriminator again right it’s a very intriguing kind of
structure and all kinds of questions should come to your mind when i am plotting this and
here is a demodulated output and here is a feedback loop which contains the vco again ok

where the center frequency of the vco is \( \omega c - \omega zero \) at the bank pass
filter as the center frequency of \( \omega zero \) ok

so we will once again call it e sub [noise] vt e sub ot and now there must be lot of
question in your mind i have a discriminator in this loop right and natural question that
will arise is if i am going to use the discriminator anyway inside the loop [noise] why
should I have this device at all right I could as well straightaway use the frequency discriminator and demodulator of the fm signal

so why all this complicate so how this works and what are the advantages of this structure over the simple straight forward discriminator are issues that will discuss in the next class thank you very much