lets now again look at the phase lock loop which we started yesterday (refer slide time: 01:06)

this is the block diagram of the phase lock loop is it has the phase detector a loop filter a loop amplifier and a voltage control oscillator (refer slide time: 01:20)

to proceed further lets assume that the input signal to the phase lock loop is an angle modulated signal of this kind it has some amplitude the carrier frequency and a phase modulation phi of t
and the let's have the also assume that the vco output is another sinusoidal signal with the
same center frequency omega c and some arbitrary phase modulation right which will
depend on what is the nature of this signal because after all the vco is given by its input
the input it has some signal

so depending on the variation of the input it will have some modulation here right

so let's start from this onwards you might notice that i have chosen the input signal to be
cosine function and the receive output to be a sine function that's deliberate and you will
soon see the reason of why i have chosen like that ok

now let's proceed further first of all how do we realize a phase detector let's first discuss
these issue a little bit and then we will look at the operation in more detail

the phase detector a very simple realization of the phase detector is as follows there are
many ways of realizing the phase detector but one of the simplest ways is to simply have
a multiplier followed by a low pass filter ok

so for example if your input here is xr t input here is e e sub vt right then the output out as
you can see the output here will contain the sum frequency component as well as the
difference frequency component when you multiply these two signals right you will get
sine into cosine and you can split it into sum frequency component and difference
frequency component right

and the difference com frequency component will be a low pass component right since
omega ct is same in both of them the difference term will continue only the difference
between the two arguments phi t and theta t and the sum will be at a frequency of center
at two omega c which will be remove by the low pass filter right

so basically if you look at the output of the low pass filter let me call this e sub dt that is a
phase detector output so i am realizing the phase detector as a set of these two blocks
right and e sub dt will be half of a sub c into a sub v which was the two amplitudes of the
two signals into a constant k sub d which is a constant of this circuit into sine of phi of t
minus theta of t do you agree with this

so this combination of a multiplier followed by a low pass filter will produce an output
which is this and as you can see it is this output e sub dt is a function of the phase error
instantaneous phase error if you consider phi of t as a instantaneous input phase and theta
t as a output phase coming from the vco right then phi of t minus theta of t is some kind
of a phase error between these two inputs and you have an output here which somehow
depends on this phase error of course it is not a linear dependence this is in this is a non
linear dependence because of a function sinusoid function that is coming to the picture
right (refer slide time: 05:56)
but nevertheless it is a dependence of course when phi of t minus theta t is small then this non linear dependence will also become almost linear dependence right

because you can approximate sine phi t minus theta t as equal to phi t minus theta t when the argument is small ok

so k sub d is a phase detector constant which is associate in fact this whole thing is a phase detector constant but k sub d is associated with the multiplier multiplier in the phase detector

they are other ways of realizing phase detector but purely this is a general principle right you can of course use other devices

so this is how the phase detector of the phase lock loop will be characterized in our model here because typically you will realize it using this combination

lets also look at how the vco can be characterized because once i know the characterization of each block in this loop then i can proceed with the analysis of this loop that’s what i am going to do right to under understanding that all of you are looking forward to

so how do we model the vco the vco can be modeled in the same way that we modeled an fm signal right because after all vco is performing the same function it will produce an output whose frequency is dep whose frequency deviation depends on the input voltage right that is the characterization of the vco its the same characterization

so what is a instantaneous frequency deviation of the signal that i have just depicted you are saying that the vco is producing an output a sub v sine of omega ct plus theta of t

so what is a instantaneous frequency deviation it is d theta by dt this must be proportional to the input signal right and if you follow the notation that i have used in the diagram of the phase lock loop input signal to the vco is e sub vt right
so $d\theta/dt$ is some constant let me call it $k_{v}$ as a vco constant times $e_{vt}$ or if you want to model the phase itself $\theta(t)$ is $k_{v}$ integral of $e_{v\alpha}$ $\alpha$ d $\alpha$ right

so these are the two equations which will model the vco right the instantaneous phase of the vco is governed by this equation which is a standard fm equation really nothing special about it (refer slide time: 08:25)

![Image of VCO equations]

so $k_{v}$ here is the vco constant of course you can work out what will be the dimensions of all of this constants that we discussed right ok

so we have now a characterization of every component that we need of course we already know how to characterize a filter and how to characterize an amplifier

so i don’t have to spend time on that the two new components that we are introduced were the phase detector and the vco which you have characterized now

so having got a characterization for each of them let us develop a mathematical model which will serve as a framework for our discussion or our understanding of the working of the phase lock model

in this mathematical model it will be nice if i can get rid of this $\omega_{c}$ $\omega_{c}$ has no direct role to play in this discussion right because $\omega_{c}$ is present in the input as well as in the output right effect for all practical purposes what is that i want i would to do

i would like to see understand how the phase lock loop produces an output whose instantaneous phase $\theta(t)$ is close to the input phase which is $\phi(t)$

so for all practical purposes i can consider $\phi(t)$ as some kind of input to the system think of this as a system whose input is $\phi(t)$ right unkn unknown phase instantaneous phase and i want to produce an output in a close loop manner such that the theta that i am introducing follows $\phi(t)$ right
so i am i will like to produce a model of that kind right so if i about to do that i can
redraw that loop that i have just discussed i have given to earlier in terms of an equivalent
diagram like this

so i am now talking about a model for the phase lock loop mathematical model which
will help us to do the analysis of course it turns out to be non linear model right
we will see why it is a non linear model very soon what is it that we have the input we
have the unknown phase phi of t right the incoming phase phi of t and i am comparing
this the phase detector compares this phase with the phase of the vco output right which
is theta of t right

and what does it do it produces an output which is of a difference of these two phi t
minus theta t and then produces the phase detector output is proportional to the sine of
this difference right

so not sign but sine right the sine function so there is a sinusoidal non linearity here this
would model your e sub dt the output here the output here will be precisely what we are
describe to be e sub dt isn’t it except that they will be also a constant which is equal to
half ac into av into k sub dt right as if there is an amplifier of this scale

just to recap if doe if it comes from please remember that this was e sub dt right so so far
i have produced sine of phi t minus theta t you have want to produce e sub dt the way i
have depicted earlier i must multiply this with this constant right

so i am as i am adding again of this much in the block block diagram this is your e sub dt
at this point following this you have the loop filter the output of the loop filter goes to the
amplifier

lets say the amplifier has some gain mu right and now what should i put between this
point and this theta t to close the loop and how will the vco be represent here the ou the
vco is not being represented a as producing an output which is sine omega ct plus theta t
it is not being modeled as it producing an output which is equal to theta t right purely a
mathematical model for what the vco is doing right

so how is theta t is related to this this is e sub vt isn’t it the amplifier output is e sub vt
how are these two related [student] through the integral relationship

so to close the loop all i have to put is an integrator here the constant k sub v do you
agree with this i think this should be fairly clear from a development that ((13:28)) right

so this is a mathematical model which we can study to understand how this works and
how precisely what it will be able to do and what it will not be able to do right ok (refer
slide time: 15:31)
now we will do our discussion in two phases in phase one we will assume that somehow the loop comes in the lock just to simplify our study right somehow the loop is locked and try to understand when the loop is locked what is a nature of e sub dt right that is phase one of our discussion

phase two of our discussion obviously should try to tell us how the lock actually happens right so should we do that will go through the discussion in two phases rather than trying to understand everything in one part

so in the first instance assume that that the pll the phase lock loop is is operating in a locked condition you can simply say that the pll is locked pll is locked which essentially means what is a meaning of this this being in a lock condition what we are trying to say by this is that theta of t is a good estimate of phi of t right that is what the lock condition means for us it may not be exactly the same but it’s a good estimate of incoming phase phi of t

this will imply in turn that the error between these two if at all there is a difference would be small right that is the meaning of the lock condition

so phi of t minus theta of t could be small in a lock condition right and if that is so we can approximate sine phi of t minus theta t by phi of t minus theta t do you agree with that

assuming that this is small now when the loop is not in lock condition we cannot say that because this may be considerably large in that case

so when the loop is not in lock condition this approximation cannot be used when the loop is operating in the lock condition it is convenient to use this approximation because what it will do is it will convert this non linear model why is it a non linear model because of the presence of this function here right otherwise everything else is a linear since linear if you look at the input output relationship it’s a linear system every block is a linear system right this is the simple amplifier this is a linear filter this again an amplifier this an integrator everything is linear right this is an adder right everything is linear except for this block
here the input is some thing and output is sine of that input is some phi t minus theta t the output is a sine i

so however if i use that approximation this also becomes linear in fact this goes this is not even required right i am removing this and that makes it a linear model for the phase lock loop

so that gives us what is called the linearized model of the phase lock loop and once you have this of course i thing you don’t have you haven’t yet gone through a cohesion control systems once you have this it becomes the very simple case of a close loop a linear feedback control system right it doesn’t matter we we will not need that background we will whatever we need we will develop it here we will use it here

so pll becomes then a linear feedback control system right and when you are working with linear feedback control systems it will you will find it is very convenient to work not only in the time domain that we are discussed in fact it’s more convenient to work in the laplace transform domain right rather than the previous Fourier domain (refer slide time: 22:08)

so very briefly will also work with the laplace transform domain all the most of the time we will not

so if i work with the laplace transform domain this linear model becomes something like this with the laplace transform domain i will represent every function that i have been so far dealing with as function of s right

so instead of phi of t at the input i will consider it laplace transform phi of s as a input ok capital phi of s is the laplace transform of the phase function phi of t small phi of t right

and here you will have theta of s it’s a capital theta of s the sinusoidal non linearity now goes right i i wont have been able to use this laplace transform notation if i had the sinusoidal non linearity because i wont know how to characterize that using the laplace
transformation right i can only deal with linear systems using Fourier transforms and laplase transforms right that’s the convenient thing to do

so that sinusoidal thing goes because of this approximation right you only have the next stage which has which is again half a sub c a sub v into k sub d followed by the loop filter followed by the amplifier let me simply call it again mu followed by the integrator the e sub vt is here ok

[student] huh yes so this should have been simply one by s right i should represent this as one by s you are absolutely right

now since we assumed that the loop is in a locked condition which means phi of t minus theta of t is small and some lets say constant value some what we basically what we are saying is phi of t is approximately equal to theta of t this would in turn imply that d phi by dt is approximately equal to d theta by dt right

but what is d phi by dt d phi by dt is proportional to the message signal mt if the input signal is a fm signal right and d theta by dt is proportional to e sub vt and since these two are approximately equal what is it mean that e sub vt would be approximately equal to the message signal mt right

so therefore this output here is my demodulated output right this is a point that at the moment you need to understand right e sub vt represents my demodulated output the point therefore i can summarize the discussion in the following way when the loop is locked since this will be so this will be so and therefore e sub vt would approximate the message signal mt and therefore the input to the vco is the demodulated output that we are looking forward looking for is that clear

so we are therefore finish the discussion on what is what i mean we have to still of course understand the mathematical analysis of this view which will do as we go along but right now even without going through any mathematics if we assume we can conclude that in the lock condition the e sub the the output e sub vt is proportional to the message signal mt that we are looking for in fm demodulator and therefore the phase lock loop in a lock condition works like an fm demodulator ok

you all with me is there any question any discussion at this stage we have to still do the mathematics right but without the mathematics this is where it is ok with the no questions lets proceed further and the next step that we have to take up take up is how does the locking actually occur right you have to understand how does a pll walk into a locked condition right

so you want like to we like to show that the phase error that we have at any instant whatever phase error we have will tend to drive the pll into a locked condition ok that’s the second stage of our discussion
we like to show that the whatever instantaneous phase error the loop has right or whatever error we have here assume be using this diagram i should be using this diagram whatever error we have here at at any time will serve will operate in such a manner that it will try to drive with serve to zero or make it small make it self small right

the loop will work in such a manner that this error will tend to become small the loop will tend to be in a lock condition remember if doing that i cannot use the linear model right because initially the phase error may not be small right

so i must now work with the at this stage work with the non linear model once a lock has been achieved we can work with a linear model right so first lets try to understand this thing

to carry out this discussion i will i will keep to simplify the discussion somewhat i will assume that the loop filter is absent right remember there is already a filter which is inbuilt into the phase detector there is a low pass filter there right that we are assuming is still there but the additional loop filter that i have put here right is not there

when the additional loop filter is not there we call this as a first order loop right as to why we call it a first order loop is something will will become obvious later but right now we will assume that this is absent (refer slide time: 29:32)

so let me state this assumption we will assume that loop filter is absent first order loop and we will see we will find that the pll works even when there is no loop filter and then will try to understand why a loop filter in what manner a loop filter might help right at the moment we will assume that the loop filter does not work does not is not present at all so this is for simplicity [noise] ok

so we have e sub vt equal to half mu times a sub c a sub v k sub t into sine of phi of t minus theta of t right and theta of t phi of t is a input theta of t is a output

let me denote this complete thing as k sub d as a constant k sub d rather than carrying on all this constants let me throw out right k sub t this t of course does not denote time you
can just this t stands for total gain you can think of this as some kind of a gain in the loop right this is a gain term this is a gain term well these are amplitude terms but effectively they are coming in this product right

so this is called the loop gain total loop gain that’s why i am putting k sub t so don’t confuse this t with time this times for total i could have probably put capital t to simplify but lets keep it small t

so k sub t integral of sine phi of t a phi of alpha may be minus theta of alpha into alpha right is there an you see where is phase is coming from you are looking at this output this is the integral of this right

now we because the loop filter has gone because the loop filter has gone e dt is nothing but equal to e sub dt except for a constant mu right

so therefore ev t is same thing as ed t except for including mu right i am able to do this because i am assume ignoring the loop filter because i am assuming that the loop filter is absent right

so these two things becomes the same except for the constant mu is that ok so that is why e sub vt is this which is this and integral of this is the theta t is that ok or alternatively i can write d theta by dt which is the model for the vco this is also model for the vco can be written as k sub t sine phi of t minus theta of t because this is from minus infinity to t and we are differentiating with respect to t

and this is actually the governing equation which dictates the dynamics of the loop you understand at any time instant phi of t is the input phase theta of t is the output phase the output phase at any time instant as a function of the input phase is governed by this non linear differential equation do you appreciate this

so this non linear differential equation which we have now arrived at provides it needs to study how theta of t will evolve as a function time right if i know how to solve this equation if i know how this equation will behave that will give me an understanding of how theta of t behaves as a function of time with respect to phi of t right that’s what i need to study

unfortunately because this is the non linear equation its also very difficult to study right so we need to these are special methods of trying to get an understanding of how this works right

to do that is any questions so far before proceed further to do the discussion further will do the discussion when phi of t is of a special is a is a special kind of function rather than for a general kind of function right

to look at a general solution of this is a very cumbersome very difficult exercise to simplify the discussion further lets assume that phi of t has a very cen special simple
nature what is it mean phi of t represents what represents a phase modulation of the input
right i am i am going to make that modulation very simple right that’s all i am saying
because study study the effect and to study how the phase lock loop actually achieves
lock right

so to do that what i am going to say is let the input fm signal let the input to the fm
modulator of the transmitter be a very simple kind of signal and the simple kind of signal
i am considering here is a unit step signal unit step function not a general m of t but m of
t is equal to ut right that is the sim simplification i am bring bringing about

so let the input to your fm modulator be a unit step function what is it mean that your d
phi by dt is well let i wont a ssume call it its its proportional to unit step function is equal
to delta omega times that is the instantaneous frequency lets say was constant lets say the
input signal was having the carrier was the fm modulator was producing a constant
frequency carrier signal initially and suddenly you provide a step change in the frequency
that’s equivalent to mt b step function is that ok (refer slide time: 34:51)

so you are producing the step change in the input signal so if you take your mt to be a
step function such that this change in amplitude of the input to the vco input to the
modulator produces a frequency change of delta omega the frequency deviation right and
now it becomes it so from fc it becomes fc plus delta f the output output fre this was mt
but now i am plotting the carrier frequency initially the carrier frequency was fc now
becomes fc plus delta f right

so this is a kind of simplification talking about so this is a step of magnitude delta omega

let me denote the phase error phi of t minus theta t by a symbol psi of t right i am going to
call the phase error

so let me define sine of t as the instantaneous phase error then i can write d theta by dt is
equal to d phi by dt minus d psi by dt right that comes from this equation but d phi by dt
is modeled by this equation right this is delta omega i am assuming i am writing the
equation only for $t$ greater than zero right so that I don’t have to work worry about the file that’s the unit step function involved at $t$ equal to zero right.

so let me write this equation only for $t$ greater than or equal to zero so this is delta omega minus $d\psi$ by $dt$ for $t$ greater than or equal to zero right.

and what is $d\theta$ by $dt$ equal to in turn remember your $d\theta$ by $dt$ its $k_t$ sine of $\psi$ t right (refer slide time: 39:53)

so this is equal to $k_t$ sine $c$ sine of $\psi$ t so what do I have now for $t$ greater than or equal to zero right basically what I have derived is this equation if you look at look at these two together right if you look at these two together I have a differential equation for the phase error right which I can rewrite as $d\psi$ by $dt$ plus i am taking this to the right hand side plus $k_t$ sine of $\psi$ of $t$ is equal to delta omega for $t$ greater than zero.

so for the special case when the fm signal is essentially a step change in a frequency at the input to the pll the governing equation for the phase error is this differential equation right and we like to understand under what conditions will this differential equation imply a movement towards $\psi$ of $t$ becoming zero right.

how the $\psi$ of $t$ becomes zero will when I want you become zero should come out by stating this differential equation right.

now when you want to study non linear differential equations one pictorial one physically appealing way is to represent this the behavior of this equation pictorially by in a in a by a diagram called the phase plane plot right.

so we like to I just see how that happens so we like to try to study this through a device called the phase plane plot.

the phase plane plot essentially is the plot of these quantities $d\psi$ t versus $d\psi$ by $dt$ versus $\psi$ of $t$ right that is at any time instant how is the derivative of the quantity related...
to the quantity itself we plot this against this ok think of this as x and this as y and plot that lets called a phase plane plot ok

so we want to plot this x axis is instantaneous phase error just you can remove the timer dependence so that it does not confuse us just plot psi or d psi by dt versus psi what is the nature of this think of this as y and this as x right

so y is equal to kt times kt x y y plus kt x is equal to delta omega and you are plotting y against x what kind of function is this this a sinusoidal function

y plus kt sine x right what is the value lets say for psi is equal to zero delta omega right and so you may have a function like that from that plot a good sinusoidal function we can plot a better one this point corresponds to delta omega

so then this value will be equal to what is the peak value of the sinusoidal function what is the peak value kt plus delta omega and what will be the value here delta omega minus kt can you see this can you all of you can see this at these points d psi by dt becomes zero right at these two points that’s the way to look at it

so this point it will be equal to at one point it will be kt plus delta omega at the other points will be delta omega minus kt right so these are the this is the phase pla phase plane plot ok

so look a study of this phase plane plot is very interesting and gives us all the insight that we need to understand how a phase lock loop locks itself right

so but before we do that is this plot alright all of you understand this ok

let me read redo redo the plot so that we can have a better discussion i hope this one turn will turn out to be better right

so this is d psi by dt versus psi this is delta omega this is k sub t plus delta omega this is delta omega minus mu sub t ok what we are saying is the following that the dynamics of the phase error will be such that d psi by dt and psi at any time instant must be consistent with respect to each other through this diagram right

if d psi by dt is this value the corresponding phase error must be this value fully it’s a solution of a equation depicted diagrammatically in some sense right

so the the values of d psi by dt as psi must satisfy this relationship which is shown in this graphical form at any point in time lets assume to start with when the input signal was a of a constant frequency that’s what we are done we assuming that the input signal was at the constant frequency and suddenly there is a step gain in the input frequency right
so when the input signal was at a constant frequency for a long time let's assume that the loop was in lock earlier

when a loop was in a lock condition where was it psi was equal to zero right so you are at this point

so assume for a discussion now that PLL was initially locked so you are at this point let's call this point b because at that point psi was zero the phase error was zero right

now what will happen that let's also understand another significance in the phase plane plot before you take the argument further

let's consider at any in at a certain point in the phase plane plot where d psi by dt is positive

now once this is so what will be the nature of psi of t as a function of time what can you say can you make any statement remember dt is time increment in time this will always be positive isn't it time only progresses forward it may never goes backward right therefore d psi will always imply the positive increment

so if this if you are in a particular point at a particular point in this phase plane plot where this is positive the derivative is positive which imply that a from this point onwards the phase error will increase as a function of time right because the derivative is positive so psi of t must increase as a function of time slope is positive right (refer slide time: 50:33)

if on the other hand so this will imply an increasing value of psi of t psi of t increases as a function of time with respect to time talking about with respect to time is this point understood is the very key point we need to understand this point when d psi by now i am talking of the temporal behavior time behavior of the signal time behavior of the phase error because d psi by dt is positive at a certain point what we are saying is that will imply that the dynamics will be searched that the phase error will increase as a function of time
similarly on the other hand when $d\psi/dt$ is negative $\psi(t)$ will decrease as a function of time right right

what is it mean that if you all in the positive side of this phase plane plot anywhere we will move along this trajectory the the $d\psi/dt$ and $\psi(t)$ always have to be related by his trajectory right you can only move along this right

but when $d\psi/dt$ is positive that is you are in the positive side of this plane positive half of this plane the phase error will try to move to the right you your trajectory will move along the trajectory to the right because because the phase error has to increase as a function of time right no matter where you are you are in a positive cycle positive side you will keep moving to the right

if you are in the negative side we will keep moving to the left along the trajectory are you with me on this [student] yes the time dependence is not obvious here but the fact that this is positive it will imply that $\psi(t)$ has to increase as the function of time and where is $\psi$ increasing to the right suppose you are here how can you we are not showing exactly what is a rate at which it moves but it has to move in this direction that’s all we are saying we are not able to say at what rate it will move but we we know we do know that’s move in the right on the right hand side to the right hand side here it will move to the left hand side because $\psi(t)$ has to decrease when $d\psi/dt$ is negative $\psi(t)$ has to increase right

so in increases from this direction to the right side decreases to the left side right that’s all we are saying so we don’t know of course at what rate it increases and at what rate it decreases but the fact that it increases means that it will move along this direction at a certain rate right precise rate from this diagram we cant figure it out

but eventually where it will go we will be able to figure out ok

if we if this discussion is ok then we can proceed forward because this is the way we understand the dynamics of a non linear system through the phase plane plot mean what i am really telling you is how a phase plane plot should be interpreted to understand the dynamics right if you understood this then we can move forward alright

now suppose initially you are at this point b then the loop is initially locked so where will you start moving now will move start going towards the right till you reach this point what happens to $d\psi/dt$ at this point it become zero so it stops to increase right if $d\psi/dt$ becomes zero it stops to increase and in some sense a lock has been achieved

but lock has been achieved will not be the zero value of the phase error but with some finite value of the phase error right but we don’t know whether it is a really a perfect ((47:58))

suppose your lets say you you are you are reached somewhere here by some chance right what will happen now will try to move to the left again to the same point right
so even if you have deviated from this either in the positive side or in the negative side will tend to return to this point

so you can think of this point lets call this point a as some kind of a stable operating point of the system right

so point a is a stable operating point right you say that this will not be a stable operating point as you can see this is a quasi stable operating point an on stable operating point because if you slightly move on either side you will move away from this point rather than towards this point right

so in [coughing] excuse me in summary in the upper half plane we will move along the trajectory on the right to the right side in the lower half plane it will move along the trajectory to the left side and that leads us to the conclusion that point a is a stable operating point of the phase plane plot right

so at this point when there is a attempt to move away from this the loop will try to return to this condition right

so point a is stable operating point as i just mentioned earlier and the implication of that is when it attempts to move away from point a and the operation attempts to move away from a even by a small amount it is forced back to point a

we can also call it the steady state operating point so what we are basically trying to say is that in this condition the loop will try to arrive to this point right when you start with psi equal to zero it will not stay at psi equal to zero because of the step change in delta omega the step change the input frequency will now try to reach a point where there is a steady state error phase error right there is a steady operating point but at that point there is no frequency error because d psi by dt is equal to zero and think of d psi t when psi if psi t is the instantaneous phase error d psi by dt is the instantaneous frequency error

so the loop would have locked itself that is the vco would have locked itself to the new frequency right there is no frequency error at this stage but that reduction of frequency error to zero is associated with the finite phase error in the process right the phase error does not remain zero right

so a steady state phase error is called the psi sub ss is some value that value is governed by this point this value this is the value of psi ss right

we will write down an expression for this later but we can also see that the steady state frequency error how much is this equal to zero in its lock condition right (refer slide time: 52:09)
so we will stop here try to study this project try to write an expression for what is the steady state phase error and then try to understand what does the loop filter achieve under what conditions lock will occur under what conditions it will not occur right we like to study all these things next time thank you very much