the next topic we will take for a discussion is network theorems i am sure that you have had acquaintance with various types of networks theorems in your earlier circuit analysis courses like the superposition theorem Thevenin’s theorem Norton’s theorem and so on

so we will have a brief review of the various network theorems and this will be as i said a review of what you are already know but perhaps it will deepen your understanding of the theorems in one or more respects in the respect of certain theorems

you may also learn a few additional theorems which you have been not exposed to in your earlier courses but the normal feature of what we are going to discuss here from your point of view

may be the application of these theorems in the laplace transform domain which is ofcourse a new feature which you would not have had in your earlier courses

now network theorems furnish a particularly simple solutions of network problems in special cases so often times the solution of a particular problem using the routine methods may enter long computation or calculations

but if you use a particular theorem which is relevant to the problem on hand then the solution may become quite simple and straight forward

so after studying the various network theorems one should develop the facility of identifying a particular theorem which fetches us the solution in the most direct and simple way from the whole lot of theorem that are available to you

so this discrimination and the facility of choosing a particular theorem relevant to the context is something very important certainly network theorems also provide us special insights into the network behavior quite often

and this is an additional reason why we study network theorems as you are aware by now there are three main realms of network analysis using the algebraic method one is the DC circuit analysis methods

then the AC circuit analysis methods then the methods using the transform diagram in the laplace transform domain all these three methods have a common set of techniques of network analysis
whether you use the loop current method or the node voltage method the principle the essential basics of the network analysis methods are the same in however in the case of DC circuits you write the equations in terms of various constants which are the resistance of the network or the conductance of the network

and the variables which are DC quantities which are constants in the case of AC circuit analysis the constants are complex constants and the variables are the phases of the voltage and currents which you are trying to solve

in the case of laplace transform technique the equations once again turn out to be algebraic but the coefficients of the various variables that you are trying to solve for are functions of $s$ algebraic functions of $s$

the impedance functions the admittance functions are the functions there of and the variables themselves which you are trying to solve for are functions of the complex variables $s$

the transform of the voltage the transform of the current as the case may be so all these in the in all the three domains whether it is a DC circuit analysis the AC circuit analysis or the transform domain essential techniques are one at the same

however the details will vary and so provide provided you pay spec special attention to the small details which are demanded by the requirements of the particular realm we can be assured that the general principles governing the network analysis are the same

consequently the network theorem that we are going to talk about also have validity in all the three domains whether the DC circuit analysis AC circuit analysis or the transform domain

so what we will do is try to give the statements of the various network theorems and try to discuss them in the most general context which is the laplace transform domain and its simplification when it is the AC circuit analysis or the DC circuit analysis will be almost evident

so let us now start with a review of superposition theorem which you are sure all of them you are familiar with already in the context of DC circuits or AC circuits

the statement of the superposition theorem would be something like this in a linear network acted upon by several sources i would say let us say several independent sources the response in any element of the network of course is the sum of the responses obtained with one source acting at a time the other sources being deactivated the other sources being deactivated
since am sure you already know about this we will not elaborate on this except ofcourse to point out that when you have a whole lot of sources acting upon a linear network the crucial statement is a linear network

in a linear network acted upon by several independent sources then if you want to calculate the response in any particular quantity in a particular element say the voltage response or current response in a particular element

it is obtained by the superposition of the responses obtained by the individual sources one each one acting at one time and when the particular source is acting all the other sources are deactivated

so we specifically mention what is meant by deactivation in a moment so comments on this is the superposition when you talk about superposition it may be you have several sources distributed into the network
so then you take one source at a time it can be superposition of several sources distributed in a network so each source you take at a time alternately the if there is a single source you can decompose that you can resolve that into several elementary sources much as we have done in the case of Fourier analysis

and consider the effect of each source at a time or superposition of the elementary sources into which a single source is resolved

so in other words it may be the spatial distribution of the sources which you have super positioned or a single source at a given location which is resolved into the sum as a sum of several elementary resources and then you find the effect of each source in turn

so that means a a temporal or if it’s a waveform there is a it may be a particular source may be resolved into several functions of time independent functions of time like a Fourier analysis Fourier so the resolution of a periodic waveform into a several sinusoids

or it could be exponential type of distribution in which you take the effect of one source at a time second comment is deactivation means voltage sources are replaced by short circuits because the voltage strength is made equal to zero
that means a particular voltage source is replaced by a short circuit current sources if there are any and when you are considering the effect of other sources a particular current source is replaced by an open circuit

current sources are replaced by open circuit in other words you are reducing the strength of that source whether it’s a voltage source or a current source to zero so when you are reducing this strength of voltage source to zero it means you are replacing it by a short circuit

if you are replacing this the strength of a current source if you are reducing the strength of a current source to zero it means its replaced by an open circuit

third point is [noise] when we are considering the effect of this various sources at a time it is only the independent sources that you should consider dependent sources if they are any they form part of the linear network

[a linear network contains linear elements and the linear elements include the dependent sources also because the dependent voltage source is linearly dependent on the controlling source controlling quantity]

therefore dependent sources must by left intact dependent or controlled controlled sources must be left intact that means you should not disturb them when the independent sources are deactivated so that is important
so if you have dependent sources those dependent sources must be allowed free play as far as computation of the effect of a one particular independent source is concerned dependent sources should not be disturbed they should be there

of course their value depends upon the particular situation in the circuit the fourth point which we should like to observe is when dealing with differential equations

when dealing with differential equations differential equation approach the superposition principle applies to a suppose you are taking up a differential equations approach to solution of a network problem and then you have several sources

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and you consider one source at a time then you would like to find out the response due to leak source in turn then you can superpose the responses to the individual sources provided the response that you are talking about is a particular integral solution

so superposition will be applicable to particular integral solutions alternately the total solution with zero initial conditions so you can superpose the responses if they are in the form of particular integral solutions

or alternately if the total solution is what you are looking for and you want to superpose this total solution due to each individual source acting at a time then the superposition principle is valid provided the network has zero initial conditions

this is because when you have a initial current in an inductor or an initial charge in capacitor it spoils the linearity of the terminal relations of the equations terminal relations between the voltage and current in an inductor or a capacitor

you may recall that all such non zero initial conditions associated with inductors or capacitors can be replaced by equivalent independent sources whether DC sources or impulse sources and the laplace transform domain analysis incorporates such equivalent sources to replace the initial conditions

therefore it turns out that the in the laplace transform domain if you replace each initial condition by an equivalent source then we are essentially dealing with a network with zero initial conditions acted upon by several independent sources including those sources which you have replaced the non zero initial conditions

therefore such a problem does not arise that means in a laplace transform domain when you have several sources including the replacement sources for non zero initial conditions you can take the effect of each individual source at a time and superpose those effects
in fact the statement here a linear network emphasizes that we are talking about a network with zero initial conditions additional points we can also observe that when you are replace when you are superposing the responses you must make sure

you must understand that the superposition principle will not be valid for super positional power powers cannot be superposed in general when you want to calculate the power in a particular element you cant calculate the power due to each individual source at a time and add up the powers and say this is the total power when all the sources that are acting up at a time

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because power is nonlinearly related to a voltage or a current as the case may be however there are special exceptions to this because in such cases the power can be superposed

exceptions are one when sources of different frequencies when sinusoidal sources of different frequencies are acting in which case because as you have seen when we talked about in fury with short um in terms in terms of a calculation of power when we are discussing the fourier series that the cross product of two different frequency terms

we will have a zero average therefore it turns out when sinusoidal source have different frequencies are acting you can calculate the power due to each source at a time each frequency source and then calc superpose them the total power will can be superposed in this context

secondly it may arise that when two sources when two sinusoidal sources of the same frequency but with a ninety degree space shift are acting so if i have one sinusoid which is sin omega t type of thing and the other is cos omega t
you can find out the power due to each individual source and add them up because it turns out that the magnitude of a plus jb is square root of a square plus b square therefore if you from that um relation you will find that when two sinusoidal sources of different frequency um same frequency but a ninety degree space shift act then also power can be superposed

um a third interesting situation arises when [noise] a DC network that is a resistor network acted upon is acted upon by voltage and current sources voltage and current sources then the total power dissipated in the network

total power delivered by the sources is the sum of powers delivered by voltage sources alone and power delivered by the voltage by the sum of power delivered power delivered by the voltage sources alone and power delivered by current sources alone
this is a very special result which is interesting that if we have several voltage sources and current sources acting upon a resistive network then you calculate the total power delivered by the voltage source acting alone

all the current sources are open circuited then let lets call that PV then find out the total power delivered by the current sources acting with voltage source as deactivated let us call that Pi then the total power is Pv plus Pi which is the superposition of these two powers

these are exceptional case special cases then lastly [noise] we might like to ask at this point what is the advantage of this superposition theorem application to superposition theorem to network analysis

one answer to this is that if you have a um a complicated network acted upon by several sources then the network method network analysis method that you have to apply may be
a general one like the loop current method and the node voltage method which requires a considerable algebra.

On the other hand, if you are taking only one source at a time, it may be that the whole network configuration may be one of the series parallel type; therefore, the analysis technique that you would employ would be one where you combine two elements in series or in parallel and analysis may be that much simpler.

So this will not be possible if you are three or two or three other sources plus two or three sources present at the same time; therefore, by using the superposition theorem, you may reduce the complexity of the network or the configuration of the network to a simpler form and the analysis may be that much faster.

So, by taking the effect taking one source at a time, series parallel reduction technique may be possible which is precluded when all this sources are acting together.

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Secondly, superposition theorem provides certain insights in the network behavior when you apply that superposition theorem; you may be able to find out certain characteristics of the network behavior using the superposition principle.

And also, you may use this for proving other results; for example, you have already seen that using superposition, we can see that the response of a linear system to a composite signal can be thought of as the superposition of the responses due to impulse functions or step functions and you have got superposition integral or the um so-called fault to integral or convolution integral.

So, superposition principle can be employed to derive some other results and get into additional insights with the behavior of the system; so with this introduction about the superposition theorem which as I said is a quick review of certain points which you already know and may refining some concepts which earlier have been exposed to.
let me work out a couple of examples um now let us now consider an example where a a voltage source $V_s$ which is four plus $\cos t$ is acting upon a series RC circuit and its given that $V_c$ zero is equal to two volts

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now if you set up you are interested in finding out $V_c$ of $t$ so if you want to find out the differential equation joining $V_c$ as the output quantity with respect to $V_s$ as the input quantity

we see that $V_c$ plus the voltage drop across the resistance must be equal to $V_s$ the current in the capacitor is $C \frac{dV_c}{dt}$ therefore the voltage across the resistance is $r$ times $C \frac{dV_c}{dt}$

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therefore you have \( R_c \; dV_c \; dt \) that is the voltage across the resistance plus the voltage across the capacitor must be equal to \( V_s \) and this can written as \( dV_c \; by \; dt \) plus one over \( R_c \; V_c \) equals \( V_s \) by \( R_c \)

so the solution for this can be obtained in the usual manner the particular integral solution for this would be of course one over \( D \) plus one over \( R_c \) operating on \( V_s \) upon \( R_c \) and using the forcing function that we know we can find out the particular integral solution

and using the initial condition finally you can arrive at the solutions total solution for \( V_c \) as four minus two point five e power minus \( t \) plus half of cosine \( t \) plus sin \( t \) that is will be the total solution for \( V_c \) of \( t \)

now let us use the principle of superposition and try to find out the solution in the classical differential equation approach itself so let us take \( V_s \) to be equal to four volts a DC source at once and then let lets take \( V_s \) is equal to \( \cos t \) as a [29:18] and superpose the effects

so if i take \( V_s \) as four volts so you have a four volts source and then you have a resistance and capacitance \( V_c \) so the cop the particular integral solution for that when you have four volts DC \( V_c \) will also be equal to four volts

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the complementary function for this \( V_c \) will turn out to be minus two e to the power of minus \( t \) therefore the total solution taking \( V_c \) zero to be two volts
so if you take the total solution this is particular complementary is also taken with that initial condition this will turn out to be four minus two e to the power of minus t volts that is the V of c Vc

now on the other hand you take the source to be a sinusoidal function cos t and then you have RC network and if you have one ohm resistance and a one farad capacitor so you find out the impedance of this

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this is cos t omega equals one impedance is one minus j that is root two at an angle of minus forty five degrees so if cos t is a driving voltage and the impedance is r r root two at the angle of minus forty five degrees it turns out that the particular integral solution is point five cos t plus sin t
that is the particular integral solution and um complementary function will turn out to be one point five e to the power minus t therefore the total solution is one point five e to the power of minus t plus point five cos t plus sin t

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now if you add these the total particular integral solution is four plus point five cos t plus sin t if you take the sum of the total solution this is four minus two e to power of minus t minus two e to the power of minus t plus one point five minus e t so therefore four minus point five e to the power minus t plus point five cos t plus sin t

this is the what you get now if you compare these two expressions with the result that you would obtain under normal calculations you will observe that four plus half cos t is sin t the particular integral solution here is the correct one

however there in the total solution you have get four minus two point five e to the power of minus t plus this where as here you get four minus point five e to the power minus t the discrepancy arose because when you are calculating this total solution you have taken the initial capacitance voltage to be two volts

when you have taken this the cap um with this the second source you have taken the initial capacitance voltage once again as two volts therefore when you are adding this up you are getting the initial capacitor voltage as four volts

and that’s what this gives when t equals zero so the total solutions will not add up because you have taken the initial capacitance voltage as two volts on the other hand if you have taken the total solution taking the initial capacitance voltage to be zero volts you would get some value
and if you replace the initial voltage across the capacitor by an additional DC source of two volts then you get the total solution correct so you can work that out i will leave the details to you

you then you take three sources now you replace the initial capacitor voltage by two volt source that means you take a four volts DC source here as before then a cos t term a resistance here R equals one ohms and the capacitor you replace it by an uncharged capacitor plus a two volt source represent the initial conditions and you take this your to be your Vc and calculate this Vc using this source at one time this source as one time and this source as another time

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you will get the correct solution even if you superpose the total solution so that is what we meant earlier that you can superpose the total solutions with zero initial conditions because now this capacitor is assumed to have zero voltage

because the whatever non-zero initial conditions was there it is replaced by the equivalent source let us take another example let us consider a second example in which you have an independent voltage source and independent current source and a dependent voltage source
so we would like to use the superposition principle to analyze this network so we consider this voltage source acting therefore four volts and you have one ohm resistance another one ohm resistance and you have a dependent source dependent source as i said should be kept intact you should not disturb that

so if this is Vi this is four times Vi and you have one ohm resistance and another one ohm resistance and this current source is open circuit so in this network we have four volts driving a two ohm resistor in this circuit therefore a current of two ampere flows through that

and consequently am sorry this should this is one volt i have one volt here therefore this should be one volt sorry this is one volt and consequently the current here is half an ampere and Vi is point five volts and four Vi is therefore is two volts

and this two volts will drive a current of one ampere through this and you will develop a voltage of plus one volt across these two terminals and that is the complete analysis of this circuit with one voltage source only acting

now we also have a current source therefore let us find out the performance of this circuit when this one ampere current source is only acting and this one volt source is replaced by short circuit

so you have circuit like this this is Vi and once again you have the dependent source four times Vi and you have one ohm resistance another one ohm resistance and you have a current source of one ampere

now let us see what happens as far as this loop of two resistance are concerned there is no driving voltage here therefore the current here is zero and the voltage Vi also is zero if
this voltage $V_i$ is zero this is also four $V_i$ is also zero which means that this one ampere current source is acting on two one ohm resistance in parallel

which means a current here must be half ampere and the current here must be half ampere so that completes the analysis of the circuit with one ohm ampere source acting alone now you can put these two results to find out what happens in the original circuit

so you can see in the original circuit the current in this loop is half ampere here and zero ampere here therefore in the original circuit also the current is half an ampere

the voltage $V_i$ let me put this is in a different color so that we understand the result that we have obtained so this is half ampere the voltage $V_i$ is the voltage here plus the voltage here

the voltage here is zero the voltage here is half volt therefore this is half volt four $V_i$ this is two volts here this is zero this is of course zero because $V_i$ is zero this is also zero therefore this will be [noise] two volts

the current across the current source the this current source drives a current of half ampere here half ampere therefore the voltage across the current source is half volt here this is half volt this is one volt

so the voltage here is one point five volts across the source and the current here this is one ampere in this direction half an ampere in the other direction therefore this is point five ampere half ampere

the current in this element is one ampere here half ampere here therefore this is one point five amperes and this must be because one ampere in this direction half ampere in other direction this current must be half an ampere in this direction

so this half ampere and this one ampere together will go as one point five amperes in this one ohm resistance so now this particular example illustrates the application of the superposition theorem when they have a dependent source also dependent source also at present

as i pointed out the dependent sources must be kept intact when you are considering the application of each independent source at a time

the next theorem that we will discuss is what is called the substitution theorem it reads like this if an element in a network is replaced by a voltage source whose voltage at every instant of time equals the voltage existing across the element in the original network then the conditions in the rest of the network are not altered
let me illustrate this first by means of simple example and then later on i will discuss this suppose i have a two volt source acting in on a circuit like this then the voltage across this one ohm resistance is now obviously one volt

so suppose i replace this by a one volt source and consider this network two volts one ohm then the current in the network everything else will remain the same here we have a one ohm one ampere current here also you have one ampere current

the voltage across this resistance here and here are the same that means the this element which had a one volts potential drop across this is replaced by a source so that is the statement of this theorem
in general you have a network here and let us say that we have an element here may be linear may be nonlinear that element we have not made in this particular step stipulation it has [noise] in the laplace transform domain let us say it has a voltage Vs

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and it has a certain current I of s now what this substitution theorem tells us is that in this network if you replace this element by a voltage source whose voltage is Vs which is the same as this

then all the other conditions in the network will remain the same suppose i have an element here whose voltage is Vk and whose current is Ik in this this will continue to be Vk and the current will be Ik

what is the justification for this justification is that if you are doing the node voltage method of analysis let us take this as datum node take this as the datum node then the node equations at all other nodes inside the network will remain the same as far as this node is concerned in the original network V of s is the solution for this node voltage now here we are stipulating that node voltage to be Vs so this node voltage stipulation is consistent with the solution that is obtained in the original network

the other node equations are the same for this particular node you are giving the solution itself as the data therefore it is consistent therefore whatever solution you have obtained here will be the same as this as far the other nodes are concerned

similarly if you are using the loop equations the other loop equations are the same in this loop equation corrs corresponding to I of s across this in particular element you have some fair example as Zs I of s something would have written
and now whatever solution you had obtained for this is now incorporated here so this 
stipulation for this loop equation is consistent with the solution the other loop equations 
are the same

therefore the solution for the other element responses will be the same this is the principle 
of the substitution theorem now there were couple of points which i would like to 
emphasize here

we don’t have a tremendous advantage in using the substitution theorem to solve the 
network problems however it turns out to be very useful in proving some other theorems 
using the substitution principle we can use several other theorems

also in some cases where the switches are opened and closed you can apply the 
substitution theorem and get [noise] special insights in the circuit behavior and also the 
total solution with fair fairly straight forward fashion

so that is the usefulness of the substitution theorem second point i would like to 
emphasize is that [noise] the element that you are replacing can be linear or non linear 
doesn’t matter what it is

a third point is when you replace for example when you in this particular let us take the 
same case here suppose i have a similar case am mean not the same case suppose i have a 
case here

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you have a two volt source acting upon a one ohm resistance so the voltage across this is 
two volts now suppose you replace this one ohm resistor by another two volt source so 
we have this resultant circuit and what is the current in this
it can be anybody’s guess because this is a particular type of circuit in which any current I is a possible solution so this network we can say is not solvable because you do not have a definite solution for this network

so we have used this substitution theorem this is the one particular solution which is equal to two amperes is also solution but that is not a definite solution for this network

so when you use the substitution theorem we must also have a proviso that the altered network must be solvable otherwise you get an indefiniteness

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you you may have end up with a situation where multiple solutions are possible there is no definite solution and such such such a circuits are said to be not solvable

so when you use the substitution theorem you must ensure that the altered network is solvable as an extension of the substitution net theorem you can also say when a current is flowing through a particular element you can replace this by an equivalent current source

so you can also have a current source I of s this is alternative form of substitution theorem you can replace this particular element by a current source whose value at every instant of time is the current in the original network
so you can replace this element by a voltage source or a current source this is also a substitution theorem which is an alternate version of the main theorem in which we said an element can be replaced by a voltage source [noise]

as i mentioned the substitution theorem gives us some special insights in circuit behavior when particularly when switches are opened and closed which we will see in later

and it is also used to prove several other theorems these are this is the special advantage of the substitution theorem so in this lecture we have started the discussion of various network theorems

i mentioned network theorems are used to simplify the analysis of networks in particular cases and depending upon the problem on hand one of the network theorems may be quite appropriate to apply it provides us a solution in a straight forward fashion

so we should develop the ability to choose the particular theorem which facilitates the solution in the most effective way for a in a given situation and we talked about superposition theorem in which we said the response of a particular element situated in a network linear network

when it is acted upon by several independent sources can be obtained by finding out the response in that element with each source acting in turn keeping all the other source as deactivated and superposing the individual responses to get the total response

we mentioned that in the case of deactivation what we mean by deactivation is that the voltage source strength is reduced to zero and a current source strength is reduced to zero which means that the voltage is replaced by the short circuit the current is replaced by an open circuit
and this is to be done for all independent sources dependent source if they are any must be kept intact they are part of the linear system

secondly we observe that when you want to find out that transient solution in a network then the superposition principle can be applied with zero initial conditions in the system or with all initial conditions replaced by an equivalent sources

and therefore the principle of superposition can be extended to the sources which stand for the nonzero initial conditions and this is what we do in the laplace transform domain where non zero initial conditions are replaced by equivalent sources

we also mentioned that the superposition principle will be can prove to be effective when we are when we take one source at a time the network configuration becomes simpler like in the series parallel structure

then we can use the the analysis with each source in term in turn by request taking request simplify it in form of network analysis provided by this series parallel structure where as if you had more than one source such a series parallel reduction technique may not be possible

we observed that power cannot be superposed in general but there are few exception which we pointed out um then we discuss the so called substitution theorem where in a given network if you so choose a particular element can be replaced by a current source or a voltage source

the current source being having a value which which is equal at every instant of time to the actual current that was existing in network original network and similarly the voltage source

then the question that you will naturally ask is if you know the current in the network then what is the purpose of using a network theorem to follow for it the answer to that is we don’t use the substitution theorem to find out the current in a network

in most of the appropriate cases we use this as a kind of artifices or tactic to prove other network theorems also in some cases it turns out that [noise] where you are opening or closing the switches replacement of an open or a close switch by a suitable current source or a voltage source as we will see will be effective in providing us a solution which is [noise] um which is a straight forward one[noise]

we will discuss other network theorems in the next lecture but before that let me mention that we a third network theorem which is called the reciprocity theorem is something which you have already discussed

and you can just read right now that if in a linear reciprocal network which consist of bilateral elements and reciprocal elements the the ratio of the laplace transform and response emolument response to the excitation laplace transform is the same when the locations of the response and the excitation are interchanged
this is true in a um network containing bilateral elements and reciprocal elements this is a something which we have already discussed when we talked when we have discussing the two port network characteristics

so we will not discuss reciprocity theorem specially here as one of the network theorems this we will assume as already being discussed with this but however some ramifications of reciprocity theorem will be discussed when we talk about [52:10] theorem at a later point of time