In the last lecture, we were considering the classification of systems and networks. We saw the difference between a static system and a dynamic system, the difference between a continuous time system and a discrete time system. We also observed the difference between a linear system and a non-linear system. Basically, we said a linear system obeys the principle of superposition that is a combination of additivity and homogeneity. We promised ourselves at the end of the last lecture that we look at some examples of the equations governing the performance of a linear system and a non-linear system, and this is what we propose to do to start with.

So we recall that a linear system satisfies the principle of superposition if it’s a continuous time system. Then if \( x(t) \) as an input gives rise to an output \( y(t) \) and \( x(t) \) gives rise to an output \( y(t) \) then a linear combination of these two inputs \( c_1 x(t) + c_2 x(t) \) for any arbitrary pair of constants \( c_1 \) and \( c_2 \) will give rise to an output \( c_1 y(t) + c_2 y(t) \).
on the other hand if we are talking about a discrete time system naturally now the signals will not be functions of a continuous variable $t$ but their functions have a discrete variable let us say $m$

then the same statement will be carried over in this domain as if $x_1(n)$ gives rise to an output $y_1(n)$ and another arbitrary input $x_2(n)$ gives rise to an output $y_2(n)$

then if the system is linear if the discrete system is linear then $c_1 x_1(n) + c_2 x_2(n)$ will give rise to a similar linear combination of the corresponding outputs

now the input output relations of a continuous time linear system will be described by a differential equation the linear differential equation and here the corresponding equation will be called a linear difference equation

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let us look at some examples

equations pertaining to linear systems

(a) four $d^2 x/dt^2$ plus three $d y/dt$ plus three $y$ is two $x$ plus five $d x/dt$

so $x$ is the input and $y$ is the output

(b) $y(n)$ plus two plus three $y(n)$ of $n$ plus one plus three $y(n)$ let’s say is six $x(n)$

this is an example of the system the equation characterizing a discrete time system which is linear
[06:03] again the coefficients or the various output terms and the input terms are constants and therefore this is again a linear difference equation with constant coefficients
so this is a characteristic of a linear discrete time system
let us take another example
four t d y by d t plus two square y equals six x plus five d x upon d t
now here we have the coefficients of the various derivative terms as the functions of time
two t two t square and so on
this also is an equation pertaining to a linear system

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[07:03] one can easily manipulate these equations find out the-, the y one which satisfies this or a given input x one a y two which satisfies this for a given input x two and combine these two to show that if x one of t gives rise to y one t x two of t gives rise to y two of t
this condition is satisfied even here
in other words if you have a linear system it does not mean that the differential equations should have only constant coefficients
even if the coefficients are functions of the independent variables which is t in this case
even then it corresponds to a linear system
so this is also an example of a linear system
a corresponding equation for a discrete time system would be something like this n y n plus one plus two y of n equals let us say five x n plus one minus six n s x n

[08:10] so you see here the coefficients are not constants but functions of n
in this location you have got n times y n plus one
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so this corresponds to the independent variable here is t
the independent variable here is n
therefore if the coefficients are functions of the independent variable still it’s a linear system it satisfies this kind of relationship and therefore these are examples of system equations which pertain to linear systems
suppose i take a non linear system
then the coefficients are not necessarily constants or functions of the independent variable
the characteristic of the differential equation corresponding to a non linear system is that the coefficients are functions of the dependent variable

[09:03] examples four y d y by d t plus six y equals two x plus three x square
so you see that y is multiplying d y by d t and also you have a x square term here and these are the two factors which destroy the linearity of this equation
similarly now y square n plus two suppose i have plus y n plus one plus three y n equals six x of n
y n plus two is multiplied by y n plus two
so you can think of this the coefficient being y n plus two multiplying y n plus two
so this is a square term that is involved here
so this fails to be a linear difference equation
so essentially to summarize what we have is in the case of a linear system whether it’s the differential equation or the difference equation the coefficients of the various terms the dependent variable or its derivative or its incremented terms like this should be either constants or functions of the independent variable t or n as the case may be

[10:20] but if the coefficients turn out to be functions of the dependent variable then it fails to be linear it belongs to the category of non linear difference or differential equations so we will leave it at that the next category of the classification that we will talk about is the difference between time in variant systems and time variable systems so we have the classification constant parameter versus variable parameters

[11:21] another name for constant parameter system is time invariant system these two are equivalent in other words the parameters which characterize the system the parameters and the various components which constitute the system are constant with respect to time take the case of an electrical network if r l and c are fixed with respect to time with respect to time it’s called a constant parameter system and on the other hand suppose you have a device in which an inductance and resistance is continuously changing with respect to time then it becomes a time variable system

[12:01] for example if i have a carbon microphone and depending upon the input signal that you are having the resistance of the microphone may be changing so that becomes a variable parameter system or a time variable system
so variable parameter system is also sometimes referred to as time variable system

generally we will be interested in talking about time variable system or time invariant system with respect to linear systems

that is because that is the main focus of our work

so if you look at these two equations here the four equations here a and b have the derivative terms with constant coefficients and this is characteristic of a time invariant system or constant parameter system

on the other hand in these two equations we have coefficients functions of time or n as the case may be

[13:05] these are time variable systems

these describe the operations of some time variable systems

so both are linear

but one these first two equations belong to the category of time invariant systems

the latter two belong to the category of time variable systems

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the importance of distinguishing between these two class of systems first of all the solutions of these equations is much more simpler than the solution of these two equations because the time factor t is involved here and further there is a very useful property of time invariant systems which i will discuss presently and that makes you that makes analysis of our systems far simpler than what it would be in the case of time variable systems

[14:05] it goes like this

if you take a continuous time system if you have an input say x(t) which gives rise to an output y(t) then suppose this x of t is delayed by or translated in time by a certain interval
say $x(t)$ minus $d$ then such an input will give a response which is similarly delayed in time by $d$ units of time no matter what $t$ you take for discrete time system you have if $x(n)$ gives rise to an output $y(n)$ and if you consider another input signal which is the same as $x(n)$ but delayed by $d$ or $k$ instance or $n$ instance $n$ minus capital $n$ where $n$ is a fixed number

[15:22] so whatever is occurring at $n$ equals one now is occurring at $n$ plus one units then the corresponding output will be $y(n)$ minus $n$ so that means the response will be shifted by the same amount but the shape of the response will not be changing at all let me illustrate this by means of some figures suppose i have $x(t)$ and this gives rise to a response $y(t)$ like this

[16:09] then suppose i delay this signal by $d$ units so instead of zero it starts here so this is $x(t)$ minus $d$ then if it’s a time invariant system we are sure that the output will be the same as this but starts a little later this is $y(t)$ minus $d$ so the same response same wave shape except it’s delayed by the same amount as the input is delayed so this is a feature of time invariant systems and this is a very useful feature as we would see later on when we talk about how you use impulse response to characterize the find out the input for find out the response for general input

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[17:03] in the case of a discrete time system let us say we have a $x(n)$ a signal like this
x_n is a function of discrete variable m and let us say this gives rise to a response y_n like this.
suppose discrete time signals will have values only at discrete points along the time axis
and so the interval between two points may be any arbitrary value depending upon the system
so let us say for integral values of n you have sequence of values x_n and the corresponding values of y_n

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now suppose i delay this signal by units
so that means it starts here
so instead of zero it starts at n
that’s your x_n
otherwise it’s the same
then the corresponding y of n will be starts not at zero but at point n here and then it is as if this whole output is translated along the x axis by n units
so that’s how the response will be occurring for a shifted input signal this particularly as you can see this particular property will be very useful in the case of when you wan to superpose the responses due to different inputs as we will see later this is all for the time invariant in the time variable case we cannot have this kind of property because as you can appreciate when this x of t is applied to the system the parameters have certain values and therefore there is certain response but if you accept t has been delayed by this amount and we are talking about the application of these at time t equals d

[19:02] at this time the parameters of the system might have undergone some change they are no longer the same values that are existing at this point therefore even though the input has the same shape the output need not follow the same shape and that’s the reason why this such property will not be valid for a time invariant system time variable system if x t gives rise to y t then x t minus d not necessarily gives rise to y of t minus d that means it’s not necessary that this gives rise to that particular response so you cannot say anything about it unless you know the actual behavior of the variables with respect to time the next property that we will talk about next classification of systems is causal systems and those which are not causal called non causal systems

[20:20] a causal system is one which is defined as follows if you have two inputs x one t which gives rise to a response y one t and x two t which gives rise to a response y two t and further
let’s say \( x_1(t) = x_2(t) \) for \( t \) less than some point \( t_0 \)
that means up to some point \( t = t_0 \)
both our inputs are the same
so you may have \( x_1 \) up to point \( t_0 \) and after wards may be it goes like this
and you have another \( x_2 \) which also follows the same waveform up to point \( t_0 \)
but afterwards may be deviates from the values which \( x_1 \) had taken

[21:35] then if this is so then for a causal system we can expect that \( y_1(t) = y_2(t) \)
for \( t \) less than \( t_0 \)
so the corresponding outputs here if \( y_1 \) had some output like these up to \( t_0 \)
and afterwards may be it goes like this

[22:02] we can say that \( y_2 \) also we have the same response may be from here onwards
that mean up to the point \( t_0 \) the same input the inputs are the same the outputs will
be the same

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such systems are said to be causal systems
now if this is a general definition of a causal system
in particular for a linear causal system if the systems is not only causal but also linear
then you can easily see if i have an input which is \( x_1 \) minus \( x_2 \) which means up to
time \( t_0 \) \( x_1 \) minus \( x_2 \) is zero
then the output must also be zero because i superpose these two this also will be zero or
we can take \( t_0 \) as equal to zero as a general case we can say if \( x(t) \) is zero for \( t \) less
than zero then the output \( y(t) \) is zero

[23:22] that’s the consequence of a linear causal system
that means whatever input you had if it’s zero up to this point and this is you \( x(t) \) your output must be zero up to this point and later on whatever output you get you have it but it must be less than zero for \( t < 0 \) that means before you apply the input you cannot get an output which is seems to be quite reasonable for physical arguments and therefore we believe that all physical systems follow this causality principle and a system which is not causal is referred to as non causal or anticipatory system

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![Image](image.png)

[24:31] this is the opposite of the causal system we believe as i said all physical systems that we can build follow the principle of causality very often one describes causality principle in these terms but that is not quite correct in the sense if \( v(t) \) have a system like this and \( i(t) \) is the response and your \( v(t) \) is the input

[25:14] so if \( v(t) \) is the input and \( i(t) \) is the response even if \( v(t) \) is zero you are having a kind of current there \( i(t) \) in that circuit
therefore we cannot say that this particular system follows this
if x of t is the input in vi voltage and the current is the y t i will exist even if y v i t is zero
therefore if you start a signal v i of t from t equals zero onwards it does not mean that the
current will be zero because the current will be driven by the belt
but this is not a linear system because of the presence of the source you do not have a
proportional relation between b and i
that is not a linear system
the systems part is linear but the source destroys the linearity of this
so this particular property is not valid for this

[26:00] but on the other hand this property will be valid for this system
if you have two voltages v i one and v i two therefore it gives the same response as long
as v i one is equal to v i two up to t equals t nought
so this is a more general definition of causality and for a particular special case of linear
causal system we can simply say if the input is zero for time t less than zero the output
will likewise be zero for time t less than zero
now another terminology that we can introduce at this stage -, it’s convenient to use such
a term causal signal is one which is zero for t less than zero
(Refer slide time [27:00])

[27:00] so a causal signal is one which is normally defined as one which is zero for t less than zero
that means this x of t is a causal signal
it's zero for t less than zero
so one can say that the property is that if a linear causal system as a causal signal as the input the output will also be a causal signal
because when we talk about transients and linear systems and so on it would be nice to have a term which describes all those signals which are zero for t less than zero we could call such signals causal signal
so we can say a causal signal given as an input to linear causal system will produce a response which is also a causal signal
we will have one more property classification which we will discuss that is the number of independent variable systems can be classified depending upon the number of independent variables that we have to reckon with in the system and this number of independent variables is also Referred to often as the dimension of the system

[28:26] so one dimensional systems
the transients in electrical circuit are one dimensional systems because t is the independent variable
for example t or n in the case of a discrete time system
let us on the other hand think of the transient that arrives at transmission line
we have a transmission line and therefore at any point x you have the voltage between these two is a function of x at time t

[29:01] so x is zero at starting “ “ let us say
therefore the voltage here depends upon two independent quantities x and t
so along the line the voltage will change and not only with distance but also with respect
to time
so you may have two dimensions
example transmission line where the currents and voltages are functions of x and t
similarly you may have a grid like structure and you are describing some parameter here
as a function of the x coordinate and the y coordinate
therefore some quantity w which is a function of the m and n two dimensions

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these are called two dimensions
next course we can continue with other dimensions also
for example you have a field problem electro magnetic field or electro static field so it’s a
function of x y z and perhaps it’s a dynamic you also have a function of time

[30:09] so it may be four dimensions and when you deal with such multi dimensional
problems in the case of a continuous system you have partial differential equations to
contend with and in the case of a discrete time system you have a multi dimensional
discrete difference equations that you have to deal with
so after having looked at the classification of various systems who would now like to say
as far as this course is concerned we will be talking about linear time invariant systems
which are causal of course and which are one dimensional because we are talking about
only one variable at a time t or n as the case may be and however we will-, and also
dynamic dynamic linear time invariant causal one dimensional systems

[31:04] this is the main focus of our course
we will also have we will be talking both about continuous time systems as well as
discrete time systems
perhaps for seventy percent of the course material we have to do with continuous time systems and thirty percent with the discrete time systems which we will take up at the end of this course.

Even though we are talking about systems in general we would be taking specific examples from electrical networks as examples of systems and the variables therefore will be voltages and currents and the independent variable will be time.

In the matter of analysis of dynamic systems electrical engineers have all had always head start over others because they developed powerful tool for the analysis of linear systems under various excitation patterns.

[32:04] The impedance concept, the phasor notation and the early application of operational methods for dealing with dynamic systems are all due to electrical engineers and further more electrical engineers had also an access to very sophisticated experimental techniques or the measurements of the various parameters under dynamic conditions high speed recording and wave form observation not only for verifying the results from theory but also to gather data which will be useful to supplement the theory with the result that the methods developed by for the solution of electrical networks can be profitably employed by for the solution of other class of networks as well.

So we often find that systems and networks of other kinds like mechanical systems or mechanical networks acoustical networks or acoustical systems hydraulic systems and so on are usually, are sometimes modeled in terms of electrical circuits the analogous electrical circuit for a given mechanical system set up and you analyze this electrical system with all the gamut or the techniques that are available for the electrical circuit analysis and once you have that you translate the results back to the original domain interpret the results suitably and then you get the solution for the non electrical system.

[33:39] For experimentation also this is convenient

So instead of having to do with large masses springs and dash box and so on you can set up a r l c circuit which is a replica which simulates the actual mechanical system and carry out all those experimentation in terms of voltages and currents which are easy to measure in the lab and then interpret the results suitably with the result domain.

[34:01] So when we electrical networks and the dynamic performance of the electrical networks in this course we have the assurance that whatever techniques we employ and whatever methods we use here can also be profitably employed for other kinds of networks and other kinds of systems.

With this we close our discussion of the introductory remarks for this course.

Next we will take up the consideration of the different signals that we come across in our discussion of linear systems and networks.

Literally a signal is a means of conveying some information but in the context of systems we take the meaning of a signal to collectively indicate the various variables which describe the status of the system at any particular point in the system or at any particular point of time.

[35:07] As far as electrical networks are concerned the variables or the signals that we deal with are the voltages and currents.
it’s the signals as I mentioned earlier which show to say give the breadth of life to a system or a network because in the absence of signals the network or system is completely lifeless and therefore an analysis of signals and knowledge of the various kinds of signals that one comes across is important for us and when we are dealing with dynamic sub systems and networks the signals are functions of time.

[36:00] we would have naturally an infinite variety of signals possible which are functions of time however a few special kinds of signals are important for us because of their simplicity for one thing later on we will also argue that any composite signal any general signal can be decomposed in terms of this elementary simple signals and therefore a study of these of these signals is of the simple signals in important for us and we will see we will first of all review some of the signals which are familiar to us already and introduce new types of signals which will be found useful in our study let us now talk about some various types of signals elementary signals that we are interested in

[37:01] one a d c signal so f of t is a constant a is independent of time so nothing further needs to be said about this so we will leave it like that then we have an a c signal what we mean by that is a sinusoidal signal so a general form of the sinusoid f of t root a cos omega t plus theta and the whole a c circuit analysis is based upon signals of this type and you know the importance of sinusoids in circuit analysis because this sinusoid has got a very distinctive property

[38:16] this is the only periodic signal which retains its shape which retains its general wave form under the linear operations of addition of two sine waves of the same frequency subtraction of one sine wave from another the same frequency of course differentiation of a sine wave will lead to another sine wave of the same frequency integration of a sinusoid will lead to another sinusoid of the same frequency so under the linear operations of addition and subtraction differentiation and integration the sinusoid retains it character the same wave shaped tails is retained so other periodic function has got this wonderful property the result therefore is that if you have a whole system with sinusoidal sources of the same frequency distributed all over

[39:04] then when a sinusoidal current passes through an inductor it produces a voltage which is sinusoid of the same frequency when a sinusoidal voltage is applied across a capacitor it produces a current a sinusoidal same frequency therefore all these currents and voltages in the entire system as for example in the power system where we have got different sources at the same frequency
all currents and voltages ideally in every point of the system are sinusoidal at the same frequency
if we did have this wonderful property then a sinusoidal current in a inductor will produce a non sinusoidal some kind of um if you have non sinusoidal current in an inductor you will have a waveform of the voltage which is completely different and if we have all this different types of waveforms mingling with each other in a complicated system we will certainly it will drive one to insanity if one has to analyze this because the waveforms are so unpredictable and so complex
so sinusoid has got a wonderful property and it stands out as i said this is the only periodic signal which has got this kind of property and we have phasor notation which deal with sinusoidal signals um analysis of systems driven by sinusoidal systems

[40:14] so we know all about this from the a c circuit analysis
so we will leave it at that except to remark at this stage that this is really a very wonderful signal

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then we have the exponential signal
so the general form of an exponential signal would be f of t is a i will put say okay a a complex number e to the power of s t where s is also could be in general complex

[41:00] a and s are in general complex
now mathematically therefore when you substitute a value of t here because s is complex and the coefficient a is complex you get a complex value for this
so this is a complex exponential signal
now what about the dimensions of s
you know from your may be from some of your earlier courses that whenever you have a
physical equation in which an exponent appears the argument of the exponent the
exponential function that is s t must be dimensionless
similarly sine omega t whatever it’s that argument must be dimensionless
so s t must be dimensionless
otherwise you can’t match the dimensions on both sides of an equation

[42:01] if s t had a dimension then e to the power of s t has one plus s t plus s square t
square by two so on and so forth
the dimensions of that you do not know what they are because if it’s s t the dimension has
some x say m or l or what ever it’s your m square l square and so on
so because of these reasons this must be dimensionless
if this is dimensionless then the dimension of s must be something per second
s t is dimensionless
therefore s must be something per second and what is the quantity which is something per
second
it’s called frequency
therefore we turn this quantity s as a frequency and because it’s in general complex we
call this a complex frequency
so s is termed a complex frequency
it’s not a frequency in the sense something is repetitive in character like in a periodic
function but because its dimensions are something per second
therefore it’s called frequency
this is called complex frequency
so this is the most general kind of exponential signal
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Then if it's going to yield a complex value for f of t what is the use for this. As it turns out that whenever we have to deal with real systems real signals any complex signal of this type will always be matched by its mate such that the sum of these two will always be yield a realton.

For example if i have a for example suppose is a complex number a e to the power of j field that is a complex number a. Suppose i put it in this form and e to the power of say s is equal to sigma plus j omega t. Such a complex signal this is the same signal. I am writing for s sigma plus j omega and for complex number a i am writing a e to the power of j field i have written this. This will always be matched by its counterpart which we have the coefficient its conjugate and likewise the complex frequency also will have its conjugate.

Usually these two come together so if you have a e to the power of s t you will also have a star e to the power of s conjugate t. These two will always occur together. So when you combine these two you observe that you have a e to the power of sigma t and e to the power of j omega t plus phi and e to the power of minus j omega t minus phi. Therefore you will have two a cos omega t plus phi. So this is what it yields. So this is the real signal.
so we don’t have to be particularly alarmed about the fact that this is going to yield a complex number because we can be sure that in analysis of any system that we are going to take up this will always be matched by a conjugate quantity of mate like this so the sum of these two will always be of this type

[45:03] now depending upon the location of the complex frequency here in the complex plane you have different kind of behavior as for the time dependence of this signal is concerned let me illustrate this here so the complex value of s can be represented in the complex plane like this with x axis representing sigma and the y axis representing omega so suppose if the value of s is equal to minus two let us say a particular location s is equal to minus two that means the value the general value of s now takes a real value this represents a signal which will be e to the power of minus two t so you will have something like this so the time dependence of that suppose let’s take a to be a real number because it doesn’t have a-, this is a real quantity

[46:03] therefore you can’t have a conjugate of this complex frequency therefore a has to be real so for example a signal like this may be three e the power of minus two t on the other hand if i have sorry s is equal to two s is equal to sorry sorry i have made a mistake here if s is equal to two here this represents an increase in signal like this an example like this three e to the power of two t
that means it’s an exponentially growing signal
if i take $s$ is equal to minus two then this is an exponentially decreasing signal may be this is three $e$ to the power of minus two $t$
on the other hand if i have two complex frequencies which are conjugates of each other like this say this value equals minus one plus $j$ two and this is minus one minus $j$ two these two frequencies together these two terms like these will give rise to a response which is of the form some $m$ $e$ to the power of minus $t$ minus sigma is now minus one cos two $t$ plus $d$

[47:33] so that means you have a decaying sinusoid
that’s how the quantity will vary with respect to time
on the other hand if i have two conjugate complex frequencies with positive real parts say one plus $j$ two and one minus $j$ two such two terms will yield a function of time which is $m$ $e$ to the power of $t$ cos two $t$ plus $p$ and that would yield a time function which is growing in time something like this
on the border line between the left half plane and the right half plane suppose i have two frequencies let’s say $j$ four and minus $j$ four these two terms will yield a pure sinusoid and we have say a typically $m$ cos four $t$ plus theta them and theta or $m$ and $p$ depending upon the coefficients complex coefficients $a$ that you are having

[49:01] so that means your complex frequency exponential signal like this encompasses in its generality the whole lot of time functions of this type
it can be a decaying exponential signal where $s$ happens to be real and negative
it can be a growing exponential where $s$ happens to be real and positive and if $s$ happens to be purely imaginary a pair of such frequencies will give rise to a sinusoidal signal pure sinusoidal signal and on the other hand if $s$ has a negative real part it indicates a decaying oscillations and if real part of $s$ happens to be positive thing it has a increasing oscillations
so we have this $e^{st}$ has as a special case the sinusoidal signals that means sum of two such exponential functions will give rise to a pure sinusoidal so we have taken now a look at three different kinds of signals which are namely the dc signal the sinusoid and the exponential signal $e^{st}$ and i mentioned that even though exponential signal in general gives rise to a complex value for a real time $t$ but two such signals can combine and will give rise to a real function of time

[50:24] so the value at any time at any point of time $t$ will turn out to be real we also have acquainted ourselves with the meaning of the term complex frequency it’s mainly because that particular coefficient of $t$ in the exponential representation of the exponential signal has the dimensions of frequency we call it a frequency it does not necessarily mean that it gives rise to a periodic quantity because none of these signals are purely periodic the characteristics of all these signals that we had talked about so far is that they are smooth functions of time that means not only they don’t have discontinuities but you can take derivatives of these signals or they continue to have continuous and therefore this is a characteristic of these signals

[51:08] so in the next lecture we will consider some examples bearing on these concepts particularly the exponential signal and the complex frequencies and we will also go on to discuss some signal waveforms which are not continuous but which are important in our further study