good morning in the last lecture we finally completed maxwells four equations we are introduce the last correction to this equations namely the displacement vector term after now we had been working in the area of statics and motional emf and that in a certain sense covers all of the theory of machines transformers generators but it misses a very very important part of electromagnetic theory and it was this part that maxwells identified when he completed this equations so the first part of the lectures iam going to just review because these kind of ideas it is good to hear them twice whatever you miss the first time you get the second times around then i will go on and look a little bit more into what we call as the wave equation so let me review what we had up to the beginning of last lecture we know that the magnetic field satisfied

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divergent B is equal to zero and we also had that curl of H is equal to j the current density we know that divergent of the displacement vector was equal to charge density and we knew that curl of the electric field is equal to minus del B by del t so this was due to the fact that B was only due to the currents and there is no such things as a magnetic charge this was from guesses law this came from amperes law and i never really proved it but i saw by motivated it by showing that it have little loops curl of H is zero except near the loop and this is paradise law and using this four laws we also showed that you can prove that electric energy is equal to one half volume integral epsilon not or epsilon into e square dv actually more accurately this is displacement vector dot electric vector and since displacement vector is epsilon E thats where this comes from you also have shown the last few lectures magnetic energy is equal to one half volume integral B
squared over $\mu \, dv$ and this came from $B \cdot H$ actually it came from $E \cdot \nabla D \cdot \nabla t$ and $H \cdot \nabla B \cdot \nabla t$ so these concept we had so what was the problem well there where three problems actually and you all have to do with this equation a first problem was there if you looked at where $B$ come from

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we know that the magnetic field is equal to $\mu_0$ not over four pi volume integral $j \times r$ one two over $r$ one two cube $dv$ so what you could do you could consider this as some kind of a integral $dB$ that is microscopic amounts of magnetic fields due to these pieces so then that would say $dB$ is equal to $\mu_0$ not over by four pi $j \times r$ one two over $r$ one two cube $dv$ and we can we are basically taking a reverse path you can now say $j \, dv$ is nothing but $j$ times $dA$ times $dl$ and then we can say that is nothing but the current times $dl$ so this tell us where the magnetic field is built up out of tiny pieces of magnetic field which are $\mu_0$ not over by four pi $dl$ cross $r$ one two over $r$ one two cubed which is where we started actually because if you remember right this is by biverse edward law well we have a little bit of current in a wire this is the little bit of magnetic field if reduces that is only one problem with this supposing we assumed that we had a wire whose length was $dl$ and have it current $i$ then this is in fact the magnetic field you will get for it so we can take the curl of the magnetic field we can see curl of $db$ and if you take the curl of this in fact you find its not zero thats a big problem this is the very building block out of which all of magnetic statics come and magnetic statics is proven that amperes law wholes and yet a very building block of magnetic statics says curl of $d$ is not equal to zero

so how can we have an equation for all of arbitrary fields yet the building block field doesnt satisfies this same equation doesnt make sense i mean after all what prevents me having little piece of wire carrying a little current a current answer to that is very obvious answer to that is the current represents moving charge so if the study current here it means there is there is charge building up at this top of this wire and this charge duplicating at the bottom of the wire so as we saw last time this little piece of wire implies at time dependent electric field the time dependent came because $Q$ is the
function of time that constant current $Q$ is linear in time studily building up in time which means the dipole field is due to a stronger and stronger dipole which means the electric field is dependent on time and if you take the derivative in time of the electric field it is independent of time

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because the electric field $E$ is equal to one over four pi epsilon not $p$ cross $i$ guess $P$ dot $i$ can never remember the formula we can check for me $P$ dot $r$ one two over $r$ cubed so you get this is the potential is $p$ dot $r$ over $r$ squared the electric will become one over four pi epsilon not $p$ over $r$ cubed ten twice cos theta $\hat{r}$ the sine theta $\theta \hat{\theta}$ hat i am not sure exactly how to put it as a vector operator so i will do it this way now this $p$ equal to the charge as a function of time times the distance $dl$ and this charge is nothing but the current times time $dl$ so this electric field is the function of time but if i take it the time derivative of the electric field where i should put partial derivative but i am being sloppy here right now one over four pi epsilon not $I$ $dl$ over $r$ cube times twice cos theta $\hat{r}$ plus sin theta $\theta \hat{\theta}$ hat the details dont matters what matters is if i take the time derivative of this dipole electric field it doesnt depend on time its a constant it depends on space of course but it doesnt depends on time
now if you look at this expression this is also independent of time but unfortunately the
curl of B is not equal to zero iam not going to prove it because there are better way of
proving it this B s dE dt is exactly the correction you need to add to this piece new times
that is required mu epsilon times that to correct this B s and make the curl of this zero so
it turns out that really this equation

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curl of H equals j is wrong is not equal to zero it is actually equal to something more and
if you ask what is that requires to make curl of the little piece of B equal to zero you find
that it is equal to j plus del D del t i mean proved it but i have motivated in that sense with
a difference between a loop carrying current i and a little piece of current carrying current
I is that charge with them in this case we already proved curl of d is zero away from the loop this case curl of V is not zero and what we find is if you add dE dt it makes curl of B zero since there is no dE dt in this case there is dE dt in this case that solve both problems and so you suggest that it should have curl of edge equal to j plus del d del t now there are two other ways of looking at this they are both given in a text book whereas this approach is not given whereas i think this is the fundamental reason ultimately after all bias award law building blocks and the bias award law does not obey ambient law you have to worry the second approach that we talked about was you take this equation and take the divergence

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shall i got divergence of curl of edge equals divergence of j but the left hand side is zero divergence of any curve is zero zero is equal to divergence of j but you already know that for any volume that current density leaving at each point integrated over the surface is that current leaving total current leaving so surface integral j dot ds the total current leaving must be equal to minus the rate at which the current is charge inside is dropping minus dQ dt but i can write this as a volume integral that is the charge decreasing means charge density at every point inside must be decreasing and this is sum of the decreases in the charge densities that is the total decreasing charge apply guesses law that gives you divergence at j is equal to minus del row del t

so i can apply that here so amperes law is telling as something is telling as that charge density is not changing thats not true i mean maxwells equations must apply everywhere including places where i am charging up or charging down an object that is in the presence of our capacitors maxwells equation must be correct and in fact it must therefore hold for the case of bias award law where i am having plus Q and minus Q building up clearly here del row del t is not zero yet amperes law requires del row del t to be zero so i need a corrective term what corrective term can i have it must be but i must add a B s it goes like del row del t in which case my answer would be divergence j plus del row del t it would become minus del row del t plus del row del t to be zero so zero equal to zero so
i need a term thats it look like plus del row del t row is nothing but divergence of D thats one of the maxwells equations so immediately tells us what we need to do as a text book point its not a proof but is being proven

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so we can just write this whole equation as the divergence of curl of H equals divergence of j plus del D del t which suggest that curl of H is equal to j plus del D del t and this is the new amperes law amperes law that includes an electric field term the third way of looking at it i want to repeat that was to look at stokes theorem and look at what stokes theorem tells us when its cuts a wire and when it goes between the plates of the capacitor thats also very well covered in most textbooks so that gave us a correction to amperes law which say that is equal to j plus del d del t
all right so now if you got this equation we can now look for solutions of it and what i did last time was to look for a particular kind of solution i say it supposing i have

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x y z and supposing i was looking for a particular kind of electric and magnetic field i was looking for an electric field E which is equal to E x of z and t then depend on x and y
along the x direction a magnetic field i will call it H which was H y which is also a function of z and t along the y direction now i can apply the various laws and look at it first apply the divergence laws what do you get divergence of E since only z is the dependence there is no derivative with respect to x there is no derivative with respect to y is equal to del Ez del z but z is zero divergence of H del Hz del z equal to zero iam going to look in a case where it s vacuums so mu is equals mu not epsilon equals to epsilon not so really when we talk about an electric and E and H iam really talking about d and B now let go to the curl equations curl of E is equal to minus del B del t

now if you look at these form which i just guess okay this is the guess i dont know if a solution x is of this type but iam going to look for it in my equations if this is the form for electric field the electric field is along x and it varies along y i mean along z so i want a curl i must draw my strokes curve in the x z plane because one of the arms of the strokes curve must be along x and the other arm must be in the direction in which the electric field is changing and therefore the normal to that strokes curve stroke surface points along y so if i take the curl of E for this particular problem what will i get i will get y hat del del z of Ex and of course there will be other terms like del del z which is zero and then the term involving x has ten z hat

but you can easily show that this is the only term that present because if you draw the strokes surface any other way we will not get any contributions so this is along y and magnetic field is any way along y its equal to minus mu along the y direction del H y del t so this gives as one equations which says del Ex del z is equal to minus mu del Hy del t the second equation we had was curl of H is equal to j plus del D del t iam working in vacuum mu not epsilon not no materials no current so it looks similar to faradays law in fact the options of current the generalized amperes law and faradays law look very similar except for the sign so i apply that same argument if i i have magnetic field in a y direction variation in z direction

so my strokes curve must be in a yz plane because one arm must point in the direction of the field otherwise you dont get a loop integral H dot dl and the other arm must point in the direction which its vary otherwise the two arm that point along H will cancel out and if you look at this three points in the x direction so we expect an x components out of this and the electric field is any way in the x component so what do we get we get x hat del del z of Hy with the minus sign thus because if we take the curl x hat del del y of Hz minus del del z of Hy Hz is zero so the only term present is minus x hat of del del z of Hy is equal to epsilon not x hat del Ex del t so again the x hats are common you can remove them and you get del Hy del z is equal to minus epsilon del Ex del t is a are very important equations come in many areas especially they come in transmission line theory you will see similar equations in power systems when we do transmission line equations okay so iam going to take this two equations and combine them and when i combine them all i do is i take the z derivative of this so i get
del square Ex del z squared which will be minus mu del del z of del Hy del t lets taking
this equations and taking the z derivative of it but because of second partial derivatives it
doesn't matter which order you do it in this will be equal to minus mu del del t of del Hy
del z but then i apply the second equation because del Hy del Z have an equation for that
so let me use that it give me minus mu del del t of minus epsilon del Ex del t so that gives
me my final equation the minus sign cancel out mu epsilon again pull out that gives me
del squared Ex del z squared is equal to mu epsilon del squared Ex del t squared its a
single equation in Ex but all of it came from a guess it came from the guess that my field
varied in the direction ninety degrees to the direction in which it is pointing both case and
if varied in the common direction z so it doesn't mean just because i got an equation there
are solutions so we have to try and solve this problem again iam going to guess i can see
that i basically want the same behavior with respect to z where i have with respect to t
otherwise the second derivatives cannot be proportional to each other so i can think a one
way which is happen which is i can say supposing Ex is equal to some function of z
minus ct so in that case if i take a derivative with respect to z or if i take a derivative with
respect to c so iam taking basically a derivative with respect to f so the second partial
derivative will be the same function so lets try it out well i will take this function and take
its derivative with respect to z
so i get \( \frac{\partial E_z}{\partial z} \) with respect to \( \partial z \) is equal to let me call this \( z - ct \) as \( u \) so it will become \( \frac{df}{du} \) \( \frac{\partial u}{\partial z} \) its that chain rule if i have a function which it depends on some variable and that variable depends on wonder iam differentiating with respect to first i can take the derivative with respect to \( u \) and then take the derivative of \( u \) with respect to \( z \) so is that it is just the chain rule but \( \frac{\partial u}{\partial z} \) is one so is equal to \( \frac{df}{du} \) if i take the second derivative \( \frac{\partial^2 E_x}{\partial z^2} \) is equal to \( \frac{d}{du} \) of \( \frac{df}{du} \) hence \( \frac{\partial u}{\partial z} \) because \( \frac{\partial^2}{\partial z^2} \) \( \frac{df}{du} \) squared is \( \frac{\partial}{\partial z} \) of this quantity again \( \frac{\partial u}{\partial z} \) is one so its equal to \( \frac{d^2}{du^2} \) of \( \frac{df}{du} \) square

now what about this respective time well i get \( \frac{\partial E_x}{\partial t} \) with respect to time is again \( \frac{df}{du} \) \( \frac{\partial u}{\partial t} \) what is \( \frac{\partial u}{\partial t} \) is \( z - ct \) minus \( ct \) so \( \frac{\partial u}{\partial t} \) is equal to \( \frac{\partial t}{\partial u} \) of \( z \) minus \( ct \) minus \( c \) so it doesnt depends on \( t \) so it doesnt give you anything but it minus \( \frac{\partial t}{\partial u} \) of \( ct \) which is \( -C \) \( C \) is a constant i assume here so \( \frac{\partial t}{\partial u} \) of \( -C \) and get minus \( C \) \( \frac{df}{du} \) let me take the second derivative \( \frac{\partial^2 E_x}{\partial t^2} \) is equal to \( \frac{d}{du} \) of minus \( C \) \( \frac{df}{du} \) \( \frac{\partial u}{\partial t} \) what is \( \frac{\partial u}{\partial t} \) is once again minus \( C \) this minus \( C \) doesnt depend on \( u \) so it can be pulled so i get minus \( C \) squared \( \frac{d^2}{du^2} \) \( \frac{df}{du} \) square so i can take all this and put it in to this equation going to do it on this screen itself but you can see all the equations in one go 

\( \frac{\partial^2 E_x}{\partial z^2} \) comes from here
it is $d^2 f_{du}^2$ is equal to $\mu \varepsilon \nabla^2 E_x \nabla^2 t$ square comes from here its minus $C$ whole square $d$ squared $f_{du}^2$ squared well $d$ squared $f_{du}^2$ square cancels out same on both sides what i get one is equal to minus $C$ squared is just $C$ squared $\mu \varepsilon$ $C$ squared or $C$ square is equal to one over $\mu E$ another way of stating the same thing is going back here i can write this as $E_x$ is equal to $f(\frac{z - ct}{\sqrt{\mu \varepsilon}})$ i had made no assumption that what so ever about what $f$ is you look at it $f$ can be anything $f$ can be a triangle or $f$ can be a square $f$ can be a trigonometric function $f$ can be a exponential $f$ can be anything all i have assume this whatever the function $f$ it depends on $z$ and $t$ through this combination $z$ minus $ct$ and i guessed that because i could see if i wanted $z$ derivative proportional to $d$ derivatives so that was to happen i should have the dependence through a combination is not just this that we can do we could have actually chosen to make this minus plus
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if you made this minus plus what would change this would still be true \( \nabla u \nabla z \) is still one this would still be true but here the minus \( \mathbf{C} \) becomes plus \( \mathbf{C} \)

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and here minus \( \mathbf{C} \) becomes plus \( \mathbf{C} \) again therefore this become plus \( \mathbf{C} \) here so if i look here i get plus \( \mathbf{C} \)
which gives me the same answer the same $C$ which means that both this solutions an arbitrary function of $z$ minus $t$ over square root of $\mu \epsilon$ and an arbitrary function of $z$ plus $t$ over $\mu \epsilon$ of both solutions of this equation which means that this guess that we made let look for the solution of this type is a good guess is that guess of given answer okay so let me summaries what you got up to right now we started where the generalized amperes law and we derive the wave equation
actually we derived it in one dimension and the wave equation looks like \( \nabla^2 E_x = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2} \) and for this problem i assume vacuum having done this i assumed that the dependence of \( z \) and \( t \) where connecting and i found that any solution of this following type \( E_x \) as the function of \( zt \) is equal to some \( f \) of \( z - ct \) or i should say \( ct \) i should say \( t \) divided by square root of \( \mu \) not epsilon not plus any other function \( g \) function of \( z + t \) over square root of \( \mu \) not epsilon not is the solution of this equation now i could have started with this same equations and i could have gone the reverse way instead of eliminating \( H \) through this equation i could have eliminated \( E \) through this equation in which case i leave that to you for a exercise

you will find that this wave equation becomes \( \nabla^2 H_y = \frac{1}{c^2} \frac{\partial^2 H_y}{\partial t^2} \) its a same equation whether you work with \( E \) or you work with \( H \) you get solve the same equation and you would find \( H_y \) of \( zt \) is equal to some \( i \) put a fiddle on top of it minus \( t \) over square root of \( \mu \) not epsilon not plus some other \( g \) \( z + t \) over square root of \( \mu \) not epsilon not so both have the same structure \( E_x \) depends on some function of \( z - t \) over \( \mu \) epsilon square root plus an arbitrary other function of \( x \) plus \( t \) over \( \mu \) not epsilon not where as \( H \) as the same form but possible different functions now the different function are not really the different functions because they are connected out we have paradise law and amperes law which means where i if i take the \( z \) derivative of \( tx \) it connects over the time derivative of \( H \),

lets try that out we have to apply this equation or we like this equation so i apply that equation what will i get i will have to take the \( z \) derivative of \( E_x \) so i will get i will have to define my symbols let me call us \( z - t \) over \( \mu \) epsilon not not as \( u \) \( z + t \) over \( \mu \) epsilon not not as \( v \).
so i will get df du times one du dz is one plus dg dv derivative with respect to z is again one that is the left hand side dx del z is equal to minus mu now the time derivative of this which is minus df twiddle by du divided by square root of mu not epsilon not plus dg fiddle dud v divided by square root of mu not epsilon not similarly if you use this equation you will get another pair of equations so clearly df f and g are related to f delta and g two we will come back later to exactly what the relation is but solving any one of this two amongst the solving both okay so now look at what kind of solution you are forming for this iam going to assume this form for electric field iam going to assume that this is Ex

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versus \( z \) at \( t = 0 \) so \( t = 0 \) I am going to assume that \( f \) is a triangle and \( g \) is a rectangle so if I draw my \( f \) and \( g \) yet say my \( f \) is a triangle which is \( f(z - t) \) over square root \( \mu \epsilon_{0} \) and it and my rectangle is this is this this is \( g(z + t) \) over square root \( \mu \epsilon_{0} \) so at \( t = 0 \) I have some electric field now what is this electric field going to do as time changes supposing \( t \) is now equal to a larger value what is going to happen is this positive value divided by a constant \( s \) means that \( z \) has to be larger in order to equal the same value of \( u \) that is \( u = \frac{z - t}{\sqrt{\mu \epsilon_{0}}} \) when \( t = 0 \) \( u \) is \( z \) \( t = 1 \) \( u \) is \( z - 1 \) over square root of \( \mu \epsilon_{0} \) so I need a larger \( z \) in order to get this same \( u \) what will happen is that for the larger for positive time right draw the same graph underneath I am drawing dotted line to show where the triangle was and where the rectangle was my triangle is going to move its going to be starting and ending later so you can see there is a shift the shift is due to the fact due to time positive time I require a larger value of \( z \) to reach the same value of \( z - t \) over \( \mu \epsilon_{0} \) not for positive time this triangle is moving in this direction what happens

if I look at the rectangle now at positive time \( z + \) something positive divided by square root of \( \mu \epsilon_{0} \) not is the argument of \( g \) so to reach the same value of this quantity I need smaller \( z \) because this is already present therefore my rectangle is going to start and end at smaller \( z \) the rectangle is moving left now the final electric field I have is the sum of the tool is that what is going to happen I have no electric field up to this point I have electric field due to this triangle this electric field drops again due to the triangle then suddenly this this rectangle this is my net electric fields a electric field is due to the sum of both triangle and rectangle as the rectangle moves in the left triangle moves right going to meet each other they are going to pass right through each other and you look at
some later time this same picture will now become a triangle moving right and a rectangle moving left

so it starts out with the rectangle moving out right triangle on the left the two of them merge and then the rectangle keep moving to the left and the triangle keep moving to the right so a very strange equation when you think about its a equation that preserves object and just lets them keep going whatever the shape i have taken deliberately two shape which adjust arbitrary one triangle and one rectangle those shapes are preserved thus another thing was noticing it is that after long enough time these thing are going to reach infinity you are going to reach however for a surface they are going to reach it and the amount of energy that reaches the surface amount of signals reaches the surface is equal to the initial signal

so this means that of previous calculations when we calculated magnetic field energy we did a volume integral j dot E and then we did integral in time we are looking at this is the power instantaneous power dissipated therefore instantaneous power introduced i am say that this thing is equal to there is a surface integral E cross H dot ds and then there was a plus the volume integral of H dot del p del t dv and we ignored this piece the E cross H piece and we kept this that was correct because coulomb law and bias evert law told as E cross H is very small for now you look at this kind of situation electric field is along x magnetic field is along y E cross H is along z and this object is moving in Z which means if i put a surface here then this object reaches here E cross H is going to be non zero and is going to be large is not going to be small therefore this term is no longer small in fact this is the term now become is very important and it is called the poynting fluxes we will come back to it sense it is a central concept in wave theory but the lesson to be learned that several lessons one is that we have a general solution possible this is the linear equation there where two kinds of solutions i could see z minus ct and z plus ct and i can use any linear combination of them because the equation was linear and i choose it some arbitrary function f of one solution plus the arbitrary function of solution g of the other solution when i graph the solution i find f and g just move they dont change shape because f is only a function of this combination whatever value of t there is just a shift so this full p s look like some z not at any given time it is a z not but the shape is the same this is a shifted shape at different time in the case of the minus sign term the shift is to the right in the case of the plus sign the shift is to the right so these are moving electric and magnetic objects and because they are moving electrical and magnetic object they are called waves will come back to more familiar waves very shortly but these are also waves is nothing un wave like about it

now the other thing that should be notice down here is we solve this problem and we got this factor so we can ask what is the significant to this factor well for that go back right here and say supposing i that t equals one at t equals zero this object z z equal u not at t is equal one where is the object well you can take it for either of this equation iam going to take negative sign case we have that z minus one over square root mu not epsilon not is equal to u not i want to solve where the u not is at t is equal to zero it was z is equal to zero not t is equal to one
it is $z - 1$ over square root of $\mu$ not epsilon not equal to $u$ not so what value of $z$
is that $z$ is equal to $u$ not plus one over square root of $\mu$ not epsilon not more generally
if I left $t b$ anything this one is $t$ and $z$ is therefore equal to $u$ not plus $t$ over this now you
have studied kinematics

so you know that if you get any solutions that says $z$ is equal to $z$ not plus some constant
times $t$ the constant is velocity what is the velocity in this case this velocity $v$ is equal to
one over square root of $\mu$ not epsilon not when you put in the numbers its not is no
accident because it is design that way you get this is equal to the speed of light in vacuum
that is its equal to three into ten to the eight meter by second so you just coming out of
the equation the equation have built in them that any disturbance that you have can be
solve for and you will get one part of the disturbance moving forward with the speed of
the light one part of the disturbance moving backward with zero plane or if you like one
moving right the other moving left this velocity is nothing but speed applied ones
maxwells saw this well I will not know exactly the history whether it is well know to
maxwells

but once we say this feature here we can realize something they realize that the entire
theory of light the entire theory of updates is just part of maxwells equations because this
is really talking about the propagation of radiation propagation of light so tremendous
kind of simplification because you dont want to have two separate theories one theory to
explain light one theory to explain electricity and magnetism same theory explains
transformers same theory explain inductors machines generators it also explains cell
phone wireless medium wave transmission machine satellite transmission it also explain
radiation from the sun it also explains all kinds of variations and its a its a enormous
simplification that resulted in physics once this additional term to amperes law was at as i
said only one thing i want to do now which is supposing instead of talking about arbitrary functions f and g supposing i had assume

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Ex is equal to some A cos of Z minus ct call it as ct if i assume this form then what do i get well i can apply this equation and what i get is del E del z is equal to minus A sin z minus ct and this is suppose to be equal to minus mu del H del t now iam going to assume Hy is also a function of z minus ct this is the function of z minus ct plus that one will also d because we know disturbance of moving as objects so that the electric field must carry the magnetic field with it magnetic field must carry the electric field with it but what is the shape of Hy well thats easy i just integrate with respect to time may be integrate with respect to time what you get it implies H is equal to A over mu times the integral of sin but this sign of u not sign of t you have to get a one over c and then cos of z minus ct this probable a minus sigh here because of z minus sign no because when i do the integration the minus sign goes away so you get that the magnetic field also look like cos of z minus ct and the electric look like cos of z minus ct and this just allow us that tremendous simplification and it allows as to do much simpler analysis of waves that is why for the rest of this course we will be look at what are monochromatic wave waves which look like sin and cosine i have continue that next lecture